

Economic Growth, Development  
and  
Exhaustible Natural Resources

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This thesis contains three essays on growth, development and exhaustible natural resources. The first essay examines the choice of occupation between productive activities and rent-seeking in an oil economy. Three regimes can occur: no rent-seeking, coexistence of rent-seeking and productive activity and full rent-seeking. An oil boom may boost GDP through a multiplier effect, or raise it less than proportional. The boom may even lower GDP depending on its impact on the allocation of talent and provision of public productive services. Booms may, however, be a curse even when the voracity effect, by which fiscal transfers grow at a higher rate than the size of windfalls, is not operative.

The rest of the thesis centres around the transitional dynamics of the Lucas-type growth models. When the ratio of physical to human capital is sufficiently high, transition paths of consumption and physical capital are U-shaped. They fall for a finite period and then rise. This distinguishes the stages of transition in the Lucas setup from that of Ramsey. We find that the oil economies, which are relatively rich in terms of physical capital, have failed to develop human capital accordingly. This capital imbalance may lead to a high but unsustainable level of consumption. It may also rationalize the negative growth effect of oil windfalls when consumption smoothing is not strong.

The third essay extends the Aghion-Howitt growth model, augmented by exhaustible resources, to address the coexistence of the steady state and transitional dynamics. We establish the existence of transition and show that the dynamics of resource extraction does not affect other sectors. The model, therefore, shares the same dynamics with the Lucas model. U-shaped path of consumption and physical capital, thus, reveals that some endogenous growth models may not produce sustainable development paths in their transition stage.

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*To my mother  
and  
the memory of my father*

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# Declaration

This thesis builds on materials that was presented during my research in different forms. The earlier versions of Chapter 2 have been presented on the following occasions:

- 2nd International Conference of *Public Economic Theory* PET 2000, Warwick University, UK 20 - 23 July 2000;
- 4th Conference in *Economics*, METU University, Ankara - Turkey, 13 - 16 September 2000;
- 57th IIPF Congress on *Political Economy of Public Finance*, Linz - Austria, 27 - 30 August 2001;
- IESG 26th Annual Conference on *Trade Liberalization, Growth and Poverty*, Gregynog - Wales, 15 - 17 September 2001.

A preliminary version of this Chapter has also been published as a joint work with my supervisor in the discussion paper series of the University of York, “Rent-seeking, Occupational Choice and Oil Boom”, September 2001, University of York *DERS Discussion Paper* 2001/11.

An earlier draft of Chapter 3 was put as a discussion paper at York, “Dynamics of Output Growth, Consumption and Physical Capital in Two-sector

Endogenous Growth Models”, December 2000, University of York *DERs Discussion Paper* 2000/62. The application of the model to explaining growth collapse of the natural resource economies was presented at the WIDER Conference on *Growth and Poverty*, Helsinki, 25 - 26 May 2001.

A very early version of the material in Chapter 4 was firstly appeared as a discussion paper at York, “Growth Constrained by Exhaustible Resources: A Creative Destruction Approach”, April 2000, University of York, *DERs Discussion Paper* 2000/22. It was then presented at the following occasions:

- 2nd international conference on *Economic Development and Environment* (EDE), Stockholm, 4-6 September 2000;
- 4th Conference in *Economics*, METU University, Ankara, 13 - 16 September, 2000;
- International Summer School on *Innovation, Technical Progress, and Environmental Policy*, Seeon - Germany, 8 - 12 September 2001.

# Chapter 1

## Introduction

This thesis elaborates the significance of a nonrenewable natural resource for development and economic growth. This is carried out from two different perspectives. First, the growth and development of natural resource-based economies and second, the case where economic growth is limited by exhaustible resources. In the former, the *endowment* of natural resources and the stream of income generated by its possession, in developing resource-based economies, is considered. Whilst in the latter, the emphasis is on the *use* of natural resources in the production process and the limitations that their exhaustibility may impose on the sustainability of growth.

The first perspective examines the growth and the level effects of natural resource endowment and resource booms. This also covers some development-related issues like rent-seeking, the allocation of talent, provision of public services and imbalance between physical and human capital in a category of economies that are heavily dependent on exhaustible resources. This in particular includes oil-abundant countries such as Algeria, Bahrain, Iran, Iraq, Kuwait, Mexico,

Nigeria, Saudi Arabia, Trinidad and Tobago, Venezuela and Yemen.

The assumed characteristics of the typical economy analyzed along the second perspective, however, more suitably reflects the features of industrial countries. Here the economy consumes the flow of the exhaustible resource as an essential input of production and enjoys a continuous and endogenous stream of technical progress. The main concern is to examine under what conditions a sustainable level of output and consumption is feasible.

Besides the Introduction and Conclusion, the thesis is organized in three other chapters. Chapter 2 considers the special features of rent-seeking in oil economies and examines the extent to which unproductive activities can explain the poor performance of resource-based economies.

The third chapter is devoted to the transitional dynamics of two-sector endogenous growth models in the tradition of Lucas(1988). The main task of this Chapter is to identify the effect of capital imbalance on the dynamics of consumption, physical capital and output growth. We apply the logic of this Chapter to address the slow economic growth of natural resource economies and in particular oil-rich ones.

In Chapter 4, by using the model of Chapter 3 as a building block, the characteristics of endogenous growth models are explored where one of the essential factors of production is nonrenewable and in finite supply. We consider transitional dynamics of the growth model developed by Aghion and Howitt(1998, ch. 5), augmented by an exhaustible resource. While sustainability of growth at the steady state is the main question of the original model, we investigate it along the transition path.

The approach taken throughout the thesis is theoretical. To give an idea



about the way which the results of our models match with the real world data, however, we simulate the model presented in Chapter 3. Apart from validation of the arguments, this allows us to proceed where analytical methods are not applicable.

Further to its theoretical merits, this thesis also rationalizes some of the aspects of poor performance of oil economies in Chapters 2 and 3. We relate our model on rent-seeking in Chapter 2 and our findings, on U-shaped path of consumption and physical capital and the dynamics of output growth, in Chapter 3 to empirical evidence on the growth and level effects of resource booms in oil economies. We do not carry out a test of any sort to match our theoretical findings with the empirical observations. We consult however empirical findings of other studies to evaluate how relevant is our arguments in the context of actual economies. This provides some “regularities” associated with performance of oil economies that can be used as “guidelines” to support the theoretical approach outlined in this study.

In the remaining parts, we describe our principal findings where the details and formal concepts are skipped.

## 1.1 Rent-seeking in oil economies

In Chapter 2, we consider an economy consisting of two sectors; the production and the endowment sector. The former, also called the *genuine production economy*, produces a final good, using labour and public productive services delivered by the government. The output of the endowment sector, on the other hand, is the flow of natural resources which is extracted from the ground. By neglecting the cost of extraction, we treat the output of this sector as a gift of nature that

does not require an input. We further assume that this output is transformed one-to-one into the final product such that their sums make GDP.

Besides engagement in productive activities, agents may choose to be rent-seeker. This requires lobbying the government to divert public expenditures away from productive services toward fiscal transfers. Political influence is more effective, the more people engage in rent-seeking and the higher is the resource rent.

We abstract from the political equilibrium arising from competition between producers and rent-seekers. We also ignore the active role of the fiscal authority itself. These hinge on two simplifications. First we assume that the fiscal authority does not have an objective of its own and solely transmits the preferences of pressure groups. In the second simplification, we rely on Becker(1983)'s concept of influence function which represents the reduced form of political competition.

The fiscal authority monopolizes the pool of natural resources and treats its output as a source of public revenues. Income tax levied from the producers is another source of public income. Public expenditures comprises public services and fiscal transfers and the fiscal budget is assumed to be always in balance.

The model is static and abstracts from intertemporal substitution of resources. In particular there is no consumption-saving choice.

The allocation of human resources between the existing activities is governed by the reward structure. Producers' wage rates are equal to their after-tax marginal productivity. Payoff to the rent-seekers, on the other hand, is the fiscal transfers net of their contribution to the rent-seeking contest. This generates a two-sided link between the reward structure and the allocation of talent.

Three regimes can occur. In the *good equilibrium*, the payoff to the produc-

ers is high enough to ensure that nobody has an incentive to be a rent-seeker. Another regime is the case of *coexistence of productive activity and rent-seeking*, where people are equally paid in both activities and have no incentive for deviation from their current career. Finally, there is a possibility for the occurrence of *full rent-seeking*, where relatively high payoffs to the rent-seekers discourage people from becoming producers.

The latter regime is not feasible in the *predator-prey* type models where predators require a minimum amount of production to grab. In our model, natural resource rent is the source of rent-seekers' income. Hence they can survive even when there is no production.

The main contribution of our study is to elaborate the level effect of natural resource booms. In order to do so, we identify the impact of the boom on different aspects of the host economy including the reward structure, the allocation of talent, the provision of public services, the output of genuine production, the level of GDP, and finally the natural resource intensity.

Based on the characteristics of the influence function, we classify the boom impacts into three mutually exclusive cases. In case one, the host economy only gains from a resource boom. It increases returns to both productive and unproductive activities, but raises the relative attractiveness of the former. For this reason a boom improves the allocation of talent in favour of productive activities and reduces the extent of diversion. A boom also, in this case, promotes the provision of public productive services. It therefore raises genuine production for two reasons. It has a positive level effect and expands GDP more than proportionally. The effect of booms on natural resource intensity is not clear in this case.

A resource boom in the second case induces both gains and losses. It promotes returns to productive activities and rent-seeking, but makes the latter more attractive. As a result, a boom in this case motivates producers to become rent-seekers. By increasing the amount of public services, the boom on the other hand boosts production. The net effect of the boom on production depends on the interaction of the negative effect of diversion of labour away from production and the positive effect of the extra public goods that the boom offers. The level effect of the boom and its impact on resource intensity is not clear in this case.

Finally, a boom may only generate losses. It raises the return to rent-seekers and lowers the pay to the producers. It therefore affects the incentive in favour of rent-seeking and supports the diversion of human resources. A boom in this case lowers the provision of public productive services. As a result, it shrinks the genuine production sector for two reasons. The overall welfare effect depends on the size of the negative effect of the boom on production sector and its positive direct effect. By reducing the size of the production economy and increasing the size of the endowment economy, a resource boom in this case changes the composition of GDP and raises the natural resource intensity.

Our treatment of the effects of a boom in this study is inclusive in the sense that it accommodates both the gains and losses of resource booms on hosting economies. In particular, our investigation captures the *horror side* of booms as an special case. We show that the *voracity effect*, by which the redistribution induced by a boom grows more rapidly than the size of boom, is a sufficient but not necessary condition for the boom to be counterproductive. A boom may result in a net loss even when the voracity is not present.

Owing to its static nature, our model is unable to address the growth effect of

booms. It does, however, generate the level effect and traces it back to the boom's impact on the allocation of labour and the provision of public productive services. This, along with the direct contribution of a boom, determines its overall welfare effect. Regarding the adverse level effect of diversion of human resources, our results accord with those of Hall and Jones(1999) and Romer(2001, sec. 3.11) who address the role of social infrastructure in level differences across countries.

Our exposition in Chapter 2 extends along the original idea of Baumol(1990) who outlines the importance of the allocation of talent rather than its pure supply for economic success and the role of relative payoffs under this allocation. Our departure point is that here there is a two-sided link between the reward structure and the allocation of human resources. The relative payoff is no longer exogenous, though there are naturally some exogenous elements in the reward structure.

In addition to the specific aspects of rent-seeking in oil economies, our results bear some similarities with the earlier contributions concerning the effect of rent-seeking on the allocation of talent; e.g. by Murphy et al. (1991), Acemoglu(1995), Acemoglu and Verdier (2000), the growth effect of this occupational choice, e.g. by Murphy et al. (1991,93), or its effect on the level of production by Murphy et al. (1993), Hall and Jones(1999) and Romer(2001, sec. 3.11). These studies in general apply a prey and predator model<sup>1</sup> where rent-seekers with a predetermined probability meet producers and appropriate part of their income. In our model, the government acts as an intermediary that taxes the producers and extracts the natural resources to finance the productive services and public

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<sup>1</sup>Grossman(1998) offers a unified framework for the interaction of producers and predators in terms of the resources that they use for defensive and offensive purposes respectively. Baumol(1990) also in a historical approach, shows how unproductive entrepreneurship has been gradually transformed from military activities to less violent forms of rent seeking.

transfers in favour of producers and rent-seekers respectively. No party plays an explicitly predatory role, though producers are underpaid due to the presence of rent-seeking.

Given the appropriation rate and the probability of inspection of producers by rent-seekers, the size of rent, in the above mentioned group of literature, is determined in terms of the producers' income. In our model, instead, the source of rent is external for individual rent-seekers but depends on the extent of lobbying. The rent is financed by natural resources and is beneficial for both producers and rent-seekers. The government is also neutral with respect to the inefficiencies generated by rent-seeking and its budget represents the influence of rent-seekers' lobbying.

In contrast with the predator-prey models, here the core of rent-seeking lies in the appropriation of the natural resources through lobbying. In this view, our model is in line with the *common pool* and *open access* models of Lane and Tornell(1996), Tornell and Lane(1998,99) and Tornell(1999). Rent-seekers in our model, due to their political influence, have the ability to extract transfers from the government that further must balance its budget. These transfers result in taxes on production and also depletion of the oil stock. Thus the power to extract transfers gives rent-seekers common access - via the government budget constraint - to the public revenues.

Chapter 2 also captures the main features of the voracity effect introduced by Lane and Tornell. We do not explain why redistribution of windfalls may exceed the size of windfall, but rationalize how it operates. In particular, our study elaborates how, in the presence of the voracity effect, a boom supports diversion and pushes the economy toward acute rent-seeking. We further show

that equilibrium switching from coexistence of both activities to full rent-seeking is likely even when the voracity effect is not operative.

The closest studies to Chapter 2, in the question of research, is Baland and Francois(2000) and Torvik(2002). The former considers the interaction between entrepreneurs and rent-seekers in a developing economy where entrepreneurs are manufacturers and rent-seekers hold import licences. The Baland-Francois's model shares with us the establishment of full rent-seeking equilibrium. Their main message is also very close to ours. The effect of an increase in the value of the economy's endowment of productive resources on the allocation of human resources between productive and unproductive activities depends critically on the nature of the equilibrium in which the economy stays. The higher the initial proportion of agents engaged in entrepreneurship, the more likely the resource boom is to favour productive activity.

The measure of entrepreneurship in the Baland-Francois model is the number of existing industrial sectors where production by domestic entrepreneurs takes place. When a good is produced domestically in a sector, it brings profit to the entrepreneur and destroys the rent of the licence holder. Rent-seeking is therefore a passive activity which is taken as residual with respect to manufacturing. The former group holds the licence for a sector unless manufacturers replace them. Paradoxically people are better off when the industrialized goods are produced domestically instead of importing them from the world market.

The effect of a resource boom on the extent of rent-seeking in Baland and Francois(2000), like ours, depends on the pre-boom situation. In our model however, a boom may support productive activities when the pre-boom level of rent-seeking intensity is low, but not necessary zero. Having noticed that a good

equilibrium is an unlikely situation in actual economies, our results seem more consistent with the empirical evidence than those of Baland and Francois.

Concerning the size effect of booms, Torvik(2002) arrives at a similar conclusion, albeit his argument is totally different. Natural resource abundance in his model increases the number of entrepreneurs engaged in rent seeking. With a demand externality, the drop in income as a result of this shift of labour can be higher than the increase in income from the natural resource. In our model there is no increasing returns in the production economy, but the diversion of public productive resources induced by resource booms amplifies the adverse size effect of misallocation of talent. This may totally offset the contribution of a windfall.

Another close study in spirit to ours is Svensson(2000) who focuses on the relationship between rent-seeking and windfalls where the latter is financed either by natural resource boom or foreign aid. Despite using a completely different model, in terms of assumptions and results, Svensson's study, where he focuses on the similar aspects of resource boom and foreign aid, shares some of our findings. Taking the exogenous component of public revenues as stochastic, Svensson shows that the mere expectation of aid may suffice to increase rent dissipation and productive public spending.

Grossman and Mendoza(2001)'s work is another attempt to understand the effect of the resource endowment on the allocation of time and effort to appropriative conflict. They find that the anticipated resource abundance amplifies the extent of diversion of human resources into unproductive activities.

Grossman and Mendoza do not consider the effect of an increase in the value of endowment on the incentive structure that subsequently encourages or discourages unproductive activities. The institutional similarity originated from natural



resource revenues and the link between rent-seeking and plenty in mining states is missing in the Grossman-Mendoza model. Neither does the political distribution of rents play a role in their model. Nevertheless their approach in reducing the intertemporal aspect of the analysis into the survival problem seems to be fruitful for one who wants to unveil the dynamic aspect of the current problem.

The present study also relates to that of Boyce(1998) who applies Becker(1983)'s political game to natural resources quota allocation. The distinction is that, Boyce studies purely the effect of a rent-seeking contest on the appropriation of an open access natural resource, while in our case rent-seekers in addition to competition among themselves, compete with the producers to appropriate natural resources as the prize of the contest.

The idea that government transfers are likely to bring about considerable rent-seeking waste is referred to in Tullock(1967)'s original paper. Following that, Katz and Rosenberg(1989) in an initial step toward what they called *the macroeconomic effect of rent-seeking* estimate the extent of rent-seeking by calculating the total change in the proportion of government spending. Our study is close to the Katz-Rosenberg idea because we consider rent-seeking in a macro framework and also trace back the effect of rent-seeking on the composition of public expenditure. The main distinction is that we consider both the causes and effects of rent-seeking. In our model, lobbying channels government expenditures toward rent-seekers and we are not interested in total change in public spending.

Another related work is Gradstein(1993) who considers transfer of rent as a by-product of the provision of public goods by a non-benevolent government. The difference in our work is the link between public goods models incorporating rent-seeking and occupational choice in natural resource economies.

Our approach is also related to that of Chung(1996) in whose paper the size of award to rent-seekers increases with the rent-seeking activities. The link in our model is however indirect and acts through the political influence of lobbying. Kahana and Nitzan(1999) is another relevant study where the potential beneficiaries of the rents attempt to influence their political allocation. The reason for lobbying is that the government is not perfectly credible and thus may not stand by its commitment and actually make the promised rent. In our case, there is no uncertainty of award but there is room for cultivation of the Treasury's interest in spending more on public transfers.

The contributions of Chapter 2 are mainly theoretical. Our findings, however, accord with the empirical evidence that are documented in the literature. We link the establishment of full rent-seeking equilibrium with the existence of acute rent-seeking in some oil economies and explain why natural resource abundant economies may be more tolerant towards rent-seeking. They have the common features of large wealth but slow growth which makes them attractive for rent-seeking. Slow growth reduces the attractiveness of innovation. Furthermore, rent-seekers are well-paid because a lot of wealth is up to be grabbed.

Our analysis also explains why some resource abundant economies lack the social infrastructure favourable to production. When rent-seeking is the dominant activity, agents lack the incentive for the accumulation of skills or the development of new goods and production techniques. In this situation "the society may also lack the 'social/political will' to change the status quo." Acemoglu(1995)

We also appeal to multiplicity of equilibria to account for cross-country differences amongst oil economies. In addition, we link our findings with a wealth of empirical evidence on the cases where resource booms are proved to have made

a net loss to their host economies.

## 1.2 Dynamics of the Lucas-type growth models

Chapter 3 explores the transition paths of consumption and physical capital and the local dynamics of output growth in the Lucas-type two-sector endogenous growth models. The novelty is that we merely rely on analytical methods and do not confine our analysis to the vicinity of the steady state.

The *Lucas assumption*, by which one sector specializes only in one type of capital, is crucial for our results. This implies that the main variables respond in an asymmetric fashion to the relative scarcity of one type of capital with respect to another. It identifies a cluster of endogenous growth models, including established R&D-based settings, that share the Lucas assumption.

The Chapter serves two distinct but related aims of this thesis. It explains the effect of bias in the portfolio of wealth of a country in favour of one type of capital. It also provides a setup that can be successfully extended to a growth model incorporating the flow of natural resources as an input of production. The former unveils some of the aspects of the development failure of oil economies that have not been covered in Chapter 2. The latter provides the required elements of our analysis in Chapter 4.

### 1.2.1 Theoretical findings

The Chapter incorporates two main contributions as follows.

**U-shaped path of consumption and physical capital** Whereas human capital accumulation enhances output growth, consumption and physical capital,

this is not the case for physical capital deepening. An increase in the relative size of physical capital may lead to a fall in consumption and decumulation of physical capital along their off-balanced paths. They firstly fall for a finite period and then rise along their transition toward the steady state resulting in a U-shaped time profile of both consumption and physical capital.

When the initial output-capital ratio is far short of its steady state, four episodes occur. First, both consumption and physical capital fall, then consumption rises but physical capital still decumulates. In the third phase, both variables rise at different rates and finally, at the steady state they both grow with a common and constant rate.

We show how the extent of consumption smoothing affect the sequence of stages of transition of consumption and physical capital. We also distinguish the situation in which consumption and physical capital fall in the Lucas model, from overaccumulation of capital in the Ramsey model.

**Local dynamics of output growth** Transition dynamics of output growth around the steady state is also studied in this Chapter. We classify the local dynamics of the rate of growth of output according to the magnitude of intertemporal elasticity of consumption. The output growth undershoots its long run value in most cases exhibiting a U-shaped behaviour. As the intertemporal elasticity of consumption falls, the minimum of output growth corresponds higher values of human to physical capital ratio. We however show that when consumption smoothing is not strong enough, output growth is increasing in the ratio of human to physical capital. The latter result is also globally valid if consumption smoothing is sufficiently weak.

### 1.2.2 Applications

We present two applications for our theoretical findings in Chapter 3.

**The growth effect of oil abundance** We apply the logic of Rodriguez and Sachs(1999) to a Lucas-type endogenous growth model. We take the stream of oil rent as a free good that facilitates accumulation of physical capital. This apparatus is compatible with regularities of oil economies that are affluent in terms of plant and equipment but are underdeveloped in terms of skill and knowledge. An unexpected oil boom with known size and duration, boosts consumption and physical capital temporarily but leaves the level of human capital intact. By creating a surplus supply in the product market, it, thus, displaces the economy away from its steady state and reduces output growth if the elasticity of substitution is large enough.

We believe that besides explanations like Dutch disease, coordination failure and rent-seeking, *capital imbalance* provides a plausible justification for unexplained aspects of poor growth performance of oil economies. The cause of failure, in this regard, lies in the high *ratio* of physical to human capital rather than overaccumulation of capital per se.

**Sustainability and optimality** Contrasting two notions of sustainability and optimality, in a concrete way, is another contribution of Chapter 3. If we interpret sustainability as the non-declining path of consumption or physical capital, our findings distinguish this concept from the sustained growth in the long-run. We show that in the presence of the latter, the former might be violated. In other words, sustained growth as described in the endogenous growth literature, is necessary but not sufficient for a non-declining trend of living standard in the

short and medium term.

The occurrence of this situation depends on the extent of the sectorial imbalance in the economy. The length of the period in which the standard of life continually declines depends on the productivity of learning technology. Hence although endogenous growth theory reconciles optimality and sustainability as two different criteria that a desirable path of development should meet, when transitional dynamics are taken into account the two issues may still be in conflict.

Endogenous growth theory provides a situation where the optimal growth path is sustainable. The analysis of transitional dynamics is, however, overwhelmed in the literature. This, in particular, needs to be taken into account where the rate of convergence is low or the economy is far from its steady state is misleading.

We claim that, although in an endogenous growth framework, the notions of optimality and sustainability are reconciled in the steady state, the former departs from the latter when the optimal paths of consumption and physical capital are declining. This happens where the long-run growth is sustained, implying that consumption and physical capital eventually rise on their transitions.

### **1.2.3 Link to the literature**

Next, we link this Chapter to the earlier contributions. Growth models with endogenous accumulation of human and physical capital are the subject of an important branch of the recent literature on growth theory. The idea presented in Chapter 3, relies on findings of Lucas(1988), Mulligan and Sala-i-Martin(1993), Barro and Sala-i-Martin(1995, sec.5.2.2), Arnold(2000) and Turnovsky(2000, ch.14),

though it is mainly built along the line of Lucas. Although a rich and growing literature has recently rooted from the seminal work of Lucas, there is still uncovered areas that bring about novelties and deserve further considerations.

Although the original idea of growth through investment in human capital was pioneered by Uzawa(1965), the reformulation of the model by Lucas is the most influential contribution in the field. In his simple setting, preferences are isoelastic, the technology of output product is Cobb-Douglas and neither physical nor human capital are subject to depreciation. Labour force grows at an exogenous rate. There exists an external effect from the social stock of human capital to the productivity of individual agents and, according to a crucial assumption, the educational process depends only on the human capital. In other words, human capital is the only asset to be allocated across sectors. Lucas only considers the balanced growth case where all level variables grow at a constant rate.

The subsequent extensions have grown along different lines. The works of Mulligan and Sala-i-Martin(1993) and Caballé and Santos(1993) are two major attempts to uncover the transitional dynamics of Lucas-type models. The major contribution of the former study is firstly in highlighting the importance of the imbalance between two types of capital and its impact on the dynamics of the model. The second contribution is to apply the time-elimination method to simulate the dynamics of the model using numerical methods. Finally Mulligan and Sala-i-Martin identify the main forces that drive the transition of the economy out of the steady state.

Caballé and Santos cover a wide range of two-sector endogenous growth models where the Lucas model, with the production of human capital using only its

previously accumulated stock, is a specific case. In their general specification, physical capital affects schooling. They also allow for the possibility of externalities from the average stocks of human capital in both sectors.

In their numerical investigation, i.e. figure 1(panels iii and iv), Mulligan and Sala-i-Martin consider the possibility of falling of consumption and physical capital during the transition period in the Lucas model. They do not however characterize the problem. We diagnose here the causes of the falling symptom completely by analytical methods.

Caballé and Santos(1993) depart from Lucas' setting by considering general forms of linearly homogenous production functions for both the consumption good and the education sector. Accounting for the dynamics of the economy in different growth regimes is the major contribution of Caballé and Santos. They identify three different regimes in which a sudden increase in physical capital may encourage or discourage investment in human capital during the transition period or leave it intact. This, in particular, illustrates that the neoclassical growth model can be considered as a special case of the Lucas setting.

Caballé and Santos also address the decline of both consumption and physical capital in an economy endowed in relative terms with a great amount of physical capital. They conclude then that physical capital and consumption in the two-sector endogenous growth model respond to the increment in physical capital in a similar qualitative way as in the Ramsey model because "economies with high ratios of physical to human capital will always decumulate physical capital, and economies with low ratios of physical to human capital will always increase their holdings of physical capital." (p.1064) Our findings, however, illustrate that decline of consumption and of physical capital in the Lucas model is different



from those of Ramsey where capital exceeds the golden rule.

Another area for the extension of the Lucas setting is to consider the case where, beside working and education, time may be spent in leisure activities. Ladron de Guevara et al.(1997) show that there could be multiple balanced paths in this case, with different growth rates. As a result, global stability would be lost and different economies may reach different steady states depending on their initial holdings of physical and human capitals. A different composition of wealth across countries therefore not only has a temporary growth effect, as in the Lucas model, it may also lead to permanent and increasing differences in income per capita. There could be multiple balanced growth paths with different rates of growth where the initial relative amount of physical and human capital determines which path is followed. The novelty of Ladron de Guevara et al.(1997) is that their consideration of leisure is more general than the specific case where the stock of human capital does not affect the marginal utility of leisure.

Besides the nonuniqueness of the steady state, the main findings of Ladron de Guevara et al.(1997) is that in the presence of leisure activities, it is more likely that an increment in physical capital from a certain steady state may discourage human capital accumulation and lead the economy to a lower steady state. This configuration accommodates the so-called voracity effect where the level of consumption grows more than proportionally in response to a sudden increase in physical capital financed by, say, a resource boom.

Bond et al.(1996) in a more general setting than those studies using explicit functional forms, establish the existence of a unique balanced growth path along which consumption, physical and human capitals grow at a common rate. Abstracting from human capital externality, they consider the case where the

education sector requires both physical and human capital.

Arnold(2000) demonstrates the global saddle stability of the balanced growth path in the Lucas framework. He also shows that some of the aspects of the dynamics of the model can be applied to the R&D-based endogenous growth models.

### **1.3 Growth limited by exhaustible resources**

Chapter 4 examines how the constraints imposed by natural resource scarcity affect economic growth and its sustainability. We take the Aghion-Howitt growth model limited by the finite supply of exhaustible resources as a benchmark. We extend their model to allow for the coexistence of the steady state and transition path, though they only focus on the former.

Concerning the steady state, we derive the closed form of the fundamentals of the model and show that constant returns to scale is a sufficient condition for the existence and uniqueness of the steady state. It is globally saddle stable and the technology parameters solely determine rate of convergence along the stable arm toward the steady state.

Incorporation of natural resources has not a net effect on the long-run rate of growth. Higher employment in the R&D sector, resulting in faster accumulation of knowledge, compensates for the adverse effect of resource incorporation. The higher the rate of resource utilization or the stronger the contribution of natural resource in the aggregate production function, the more skilled labour is devoted to R&D and hence the larger is the gap between the rate of innovation and economic growth. This happens because, by assumption, exhaustible resources do not limit the innovation process and the accumulation of knowledge. As a

result, the supply of individual brands falls with the rate of utilization of resource, while the total supply of intermediates grows at the balanced rate of growth.

The main contribution of the model lies in the transitional dynamics of the economy where, it may be optimal for consumption and physical capital to fall for a finite period along the transition path. This occurs while growth is sustained at the steady state and the sustainability condition, suggested by Aghion and Howitt, is met. Our numerical exercise suggests that the threshold for the occurrence of this situation is when the average productivity of capital is less than a quarter of its steady state. A similar exercise based on other set of parameters shows that the corresponding threshold might be even closer to the steady state, suggesting that the violation of sustainability is likely even in the vicinity of the steady state.

## 1.4 Keywords and JEL classifications

The distribution of the contents of the thesis, according to the *Journal of Economic Literature* classification, is reported in table 1.1.

The key words used in each chapter also follow.

**Chapter 2** Oil economies; Rent-seeking; Public transfers; Allocation of talent; Political influence; No-activity equilibrium

**Chapter 3** Lucas-type growth models; Transitional dynamics; Growth failure; Oil economies; Sustainability

**Chapter 4** Aghion-Howitt growth model; Exhaustible resources; Steady state; Transitional dynamics; Sustainability

Table 1.1 JEL clasifications of the Chapters			
JEL Classification	Chapter 2	Chapter 3	Chapter 4
C62		✓	✓
D72	✓		
H59	✓		
J22		✓	
J24	✓		
O13			✓
O41		✓	✓
O53, O54, O55	✓		
Q32			✓
Q33	✓		

## Chapter 2

# Rent-seeking, Occupational Choice and Oil Booms

*How can a repeated pattern be explained when it occurs across countries as dissimilar in regime type, social structure, geostrategic location, culture, and size as Iran, Nigeria, Mexico, Algeria, and Venezuela?*

Karl(1997, p.8)

*We do not have to wait patiently for slow cultural change in order to find measures to redirect the flow of entrepreneurial activity toward more productive goals ... it may be possible to change the rules in ways that help to offset undesired institutional influences.*

Baumol(1990, p. 919)

## 2.1 Introduction

Most natural resource-abundant economies underperformed resource poor economies. Growth collapse, trade deficit, capital flight, public debt, and low production are amongst the most important aspects of their failure. The puzzling fact that natural resource richness may bring about poverty is vastly documented, e.g. by Sachs and Warner(1995) and Auty(1998).

The case of oil economies seems more severe where resource rent is accrues mostly to the government; the resource rent is not diffused across the economy; and its trend is subject to high volatility. These features distinguish oil economies from other natural resource economies. The development failure of the latter group is well documented by Gelb(1988), Karl(1997) and Amuzegar(1999).

Further to the popular and established theories like Dutch disease, rent seeking has proved to be able to offer a plausible explanation for the symptom. Lane and Tornell(1996), for example, show that in an economy suffering from pressure groups who have open access to the productive assets, a resource boom may lower the rate of return on investment and thus growth. The link between natural resource endowment and rent-seeking intensity however has not been well captured in the literature. The current study addresses why some natural resource abundant economies tolerate a high level of rent-seeking.

Another motivation of this study originates from the fairly similar response of oil economies to oil booms. The similarity of performance of dissimilar economies receiving windfalls may suggest that the access to a very specific source of public revenues initiates institutions and shapes preferences in such a way that it makes some choices more attractive and less costly than others. This influences policy makers and other players to opt for a set of choices that, in the absence of natural

resources, might be far from the optimum selection. In other words, the access to a specific source of revenue makes some *economically irrational* choices, *politically rational*; Svensson(2000, p.455).

The aim of this study is to bridge notions of rent seeking and natural resource intensity, for examining the impact of the latter on the allocation of labour and the level of GDP. By introducing the central role of the state in windfall spending, we tailor our study to the case of oil economies. Besides justification of acute rent-seeking, we provide an explanation for the case where a resource boom causes misallocation of labour and diversion of public expenditures, lowers GDP, and changes the composition of GDP in favour of natural resource intensity.

We examine the endogenous allocation of human resources between productive activities and rent-seeking in a natural resource abundant economy where the state has a monopoly right on the extraction and liquidation of the natural resources (e.g. the case of oil in the Middle East and North Africa) and natural resource royalty is the major source of public revenue. Besides direct predation on producers, rent-seekers claim oil rent via a fiscal process that effectively allows open access to the public revenues. They influence the political distribution of resource rents via their lobby. Using this framework, we characterize conditions under which each type of equilibrium occurs, explore how the extent of rent in each regime is determined and how it depends on lobbying activities.

In studies concerning the misallocation of talent, two types of equilibrium are identified, namely the *good* equilibrium without rent-seeking and the coexistence of rent-seeking and production. We show that when natural resources are taken into account, another regime is feasible where there is no productive activity and all agents are rent-seekers. Rent-seeking in this case survives even when there is

no production to be taken and the economy relies mostly on oil rent.

Although a resource boom naturally rewards both productive and unproductive activities, its gain in general is not neutral. Whether a boom favours productive or unproductive activities depends on the characteristics of the pre-boom situation and the extent of fragmentation of the fiscal system. In the case where a boom makes rent-seeking more attractive, it is possible that, due to the extent of the diversion induced, the overall effect of a boom is counterproductive. Moreover a boom of sufficient magnitude is likely to displace the economy from one equilibrium to another. In this case the economy may get trapped into a high rent-seeking equilibrium meaning that the after-effects of the boom are irreversible even when the oil rent returns to its pre-boom level.

The empirical prediction of our model is that, in natural resource economies, where resource rent is spent via fiscal channels, there is a positive association between rent-seeking intensity and the extent of access of the fiscal claimants to the government's budget constraint. A resource boom, thus, corresponds to increased rent-seeking if it raises access to the fiscal budget.

The Chapter proceeds as follows. Next section lays out the empirical evidence associated with rent-seeking and oil booms. In section 2.3 the formal skeleton of our analysis is introduced. Some of the immediate results of the model concerning costs of rent-seeking follow in section 2.4. Section 2.5 characterizes the equilibrium of the model and section 2.6 analyses the effect of an oil boom on the reward structure and the allocation of human resources. This also considers the level effect of booms and their effect on the composition of GDP. These two sections contain most of the important results of this Chapter. We finally conclude in section 2.7.



## **2.2 Empirical discussion**

The approach taken in this Chapter is theoretical. By gathering some empirical findings from other studies however, we evaluate the relevance of our arguments in the context of actual economies. This, helps us to arrange some “facts” associated with the performance of oil economies that can be used to provide the motivation for the model we develop in the next section.

Putting rent-seeking at the heart of the analysis of this Chapter however, limits the link between the findings of the empirical studies and the predictions of the theoretical models. The applicability of the former is limited because these findings “do not have any direct statistical measure of employment in rent seeking activity”(Berthélemy et al. 2000, p.223). This implies “all attempts to find an accurate proxy of human capital engaged in rent seeking are bound to be imperfect.”

Bearing in mind the lack of precision inherent in obtaining such estimates, we consider the empirical observations of performances of oil economies in this section and try to lay them out in the format of “stylized facts”.

### **2.2.1 Acute rent-seeking**

The level of unproductive activities in the regions where oil economies are mainly located gives an idea about the possible effect of oil abundance on occupational choice. Berthélemy et al. estimate that amongst the main macro regions, the waste of human resources due to rent-seeking in the Middle East and North Africa (MENA) is very high. These regions at the same time suffer from a relatively low level of human capital. Since the oil economies are densely located in these areas, one may attribute the high level of rent-seeking there to the reliance of

the fiscal system on oil revenues.

The findings are reported in table 2.1 where the first row refers to the average years of schooling and the second one to the estimated percentage of human resources engaged in rent-seeking activities.

Table 2.1 The extent of rent-seeking intensity in macro-regions

<i>Region</i>	<i>OECD</i>	<i>Asia</i>	<i>L.America</i>	<i>MENA</i>	<i>S.S. Africa</i>
Total human capital	9.2	5.5	6.4	4.4	4.4
Rent-seeking intensity	25.1	20.8	22.0	38.8	34.8

Source : Berthélemy et al.(2000), table 6.2

As a case of acute rent-seeking, Auty(2001, p.83) takes as an example Saudi Arabia where, in 1994, according to his calculation, “the public sector employed 95% of the national workforce and public sector wage absorbed one-fifth of GDP, twice the ratio for Mexico and four times that for Indonesia. Jobs in manufacturing ... were left to immigrant workers who held 95% of private sector jobs in the late 1990s.”

In Trinidad and Tobago, Lane and Tornell(1996) report that public spending, due to windfalls adjustment, expanded rapidly in the late 1970s in the form of increased public-sector employment, from less than one-quarter of the labour force to more than one-third, rapid growth in public-sector wages, grants and soft credit.

Gelb(1988, p.271) also shows that in Trinidad and Tobago, public sector employment expanded in 1974-78 from 86,000 to 158,000, or from less than one-quarter to more than one-third of all workers. Unemployment also fell, but not because of labour demand from the private sector.

Taking public sector employment as a proxy for rent-seeking, one may conclude from these observations that oil economies are subject to a high level of

rent-seeking.

### 2.2.2 Divergent performances

Murphy et al.(1993) and Acemoglu(1995), who established models of rent-seeking with multiple equilibria, argue that such multiplicity provides an account of divergent cross-country experiences<sup>1</sup>. Baland and Francois(2000) show how a handful oil economies have successfully taken advantage of oil booms to constitute a sound industrial base, whereas others mostly channeled windfalls toward (public and private) consumption and rent-seeking. They argue that the marked differences between performance of some oil economies after the late 1970s oil boom, reveals two opposite patterns which are related to the pre-boom situations (*industrial base* in their setting) of the underlying economies.

They classify the above countries into three categories. The *failures* include Nigeria, Saudi Arabia, Kuwait, and United Arab Emirates. The *near failures* incorporate Venezuela, Mexico, Algeria. Finally the *successes* are Indonesia, Malaysia, and Norway.<sup>2</sup> Whereas in the failures, the share of public consumption in GDP, as a result of windfall spending, has increased, the opposite is true with the successes. The share of manufacturing in GDP almost mirrors this pattern.

Despite their very dissimilarities in size, political regimes, degree of openness, etc., the failures performed very similarly in facing oil windfalls. This might suggest that the type of fiscal revenue initiates some institutions that rewards some activities more. This gradually shapes the performances of both policy

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<sup>1</sup>Graham and Temple(2001) offers a broader study to explore the international variation in output per worker levels by appealing to multiplicity.

<sup>2</sup>Gelb(1988) explains in details the reaction of Indonesia, Nigeria and Venezuela, to the first and second oil booms in chapters 12, 13 and 15 respectively.

makers and labour force, which in turn affects institutions and policies by itself.

These observations suggest that there is a substantial difference in the response of oil economies to oil booms. Although a few of them gained from booms, most of them failed. Furthermore, the pre-boom level of rent-seeking may have an explanatory power on how the economy reacts to the boom.

### **2.2.3 Diversions created by booms**

Using corruption as a proxy for rent-seeking, Svensson(2000) finds that the mere effect of natural resource intensity on rent-seeking, amongst 66 aid recipient countries is not significant, while in countries suffering from competing powerful social groups, there exists a positive and significant association between the share of exports of primary products in GDP and the level of rent-seeking.

This is very close to Murshed and Perälä(2001)'s findings that: "natural resource endowment itself ... [has] not necessarily a negative influence on growth, but rather ... its coexistence with the lack of social cohesion can lead to disastrous growth outcomes." (p.15)

As evidence for the drift of windfalls toward diversion, Gelb(1988) reports that in response to the first oil price rise, the average pay for civil servants was doubled in Nigeria. "[C]ommentators have interpreted this as an attempt by General Gowon to stay in power ... by giving a pivotal sector of the population a sizeable share of oil wealth."(p.241).

He also explains the flow of surplus from rural to urban areas and from farmers and other rural producers to bureaucrats and militants. Gelb shows how the relative neglect of agriculture in Nigeria was increased by the dominance of oil business on conventional economic activities.

One may conjecture from these observations that oil abundance, coupled with the presence of powerful social groups, favours the diversion of human resources towards rent-seeking.

#### **2.2.4 Excessive spending**

There are some evidence that oil windfalls may coincide with an excessive rise in the level of public and private consumption in host economies. Tornell and Lane call this phenomena the voracity effect. By comparing the extent of change in fiscal transfers, stimulated by booms, with the size of booms one may examine the occurrence of this effect.

Transfers in actual economies can take several forms including holding down the rate of inflation by consumer subsidies and support price controls on consumer goods, particularly by lowering oil prices for domestic consumers, producer subsidies to supporting loss-making (usually public) firms by providing loan and required inputs, public employment and the direct transfer of oil rents to specific sections of society.

There is no hard evidence to assess the distributional impact of such transfer programmes. Gelb(1988, ch.7)'s considerations shows that the pattern of windfall spending in his sample (Indonesia excluded) is urban biased. Concentration of public investment in some gigantic projects is another sign, showing that the beneficiaries of those projects were limited. The fiscal adjustment to windfalls indicates that public transfers financed by oil rents were distributed unevenly and the more powerful claimants benefitted more. Although the distributional impact of such an spending pattern is not our concern, the stimulating effects of such programmes on rent-seeking *is* within the scope of this study.

The new component of public revenues, in the form of an enormous windfall that accrues directly to the state, inflates goals and the expectations of policy makers and other powerful agents who have access to the fiscal budget. In Karl(1997)'s words, "all oil-exporting states had the same response to the petrodollar influx: they massively increased their government expenditures." (p.25)

She also describes *state spending* as the central component of the *boom mentality*: "In 1973-74, Iran's government expenditures leapt a full 58.3 percent in real terms over the previous year; Venezuela's jumped 74.5 percent; and Nigeria's 32.2 percent ... all states embarked on huge state-led plans, financed through both petrodollars and foreign borrowing." (p.25).

Gelb(pp.22-23) estimates that approximately half of the oil windfall was used for domestic investment that was overwhelmingly public. He also reports that between 1974 and 1978, subsidies for unprofitable firms and lower income groups expanded rapidly. They grew at a rate twice that of GDP. According to his estimates on a sample of six oil exporters, "between 1970-72 and 1974-78 fiscal subsidies and transfers expanded, on average, twice as rapidly as nonmining GDP; between 1974-78 and 1980-81 they rose 1.6 times as rapidly." (p.118).

The bias towards ambitious macroprojects in heavy industries, partially financed through foreign borrowing, severely deteriorates the current account of the oil exporters at the very heart of the booming period. As a result "in a mere four years, the capital-deficit oil exporters [including Algeria, Indonesia, Iran, Nigeria, Venezuela] moved from a combined current account surplus of almost \$24 billion (1974) to a deficit of over \$14 billion (1978)." (Karl, p.27). She also reports that by 1980, "these countries showed a combined debt of almost \$100

billion, up from \$19.5 billion before the first oil boom.” (p.30)

According to Gelb(1988, p.108), out of the nineteen developing countries that invested the most in projects exceeding \$100 million each, all but five were oil exporters. Some of the indicators of public investment in macroprojects in a sample of oil-exporting countries is reported in table 2.2.

Table 2.2 Macroprojects in some oil economies

<i>Country</i>	<i>No. of projects</i>	<i>Tot. cost (b\$)</i>	<i>Av. cost (m\$)</i>	<i>Tot. cost / 1980 GNP</i>	<i>Tot. cost / 1980 windfall</i>
Iran	108	119.6	1107	1.57	10.2
Algeria	69	38.7	561	1.07	4.2
Venezuela	27	27.4	1015	0.51	5.4
Mexico	59	26.0	441	0.18	5.1
Nigeria	19	14.4	758	0.17	0.9
Indonesia	44	14.4	327	0.23	1.1
Trinidad & Tobago	7	6.9	983	1.35	4.5

Source: Gelb(1988), Table 7-4.

The table reveals that Iran initiated 108 macroprojects averaging more than \$1b each, with a total capital cost more than ten times its 1977 oil windfall and 1.5 times of its GNP in 1980. The corresponding figures for other countries show to a lesser extent the tendency for oil-led investment in macroprojects where the size of investment far exceeds beyond the magnitude of the windfall itself. The large projects identified e.g. in Algeria, Venezuela, Mexico, and Trinidad and Tobago represented four or five times the 1980 oil windfall of these countries.

The disappointing outcome of these projects is explained in detail by Gelb

(1988) who reports: “some public manufacturing projects have been unable to cover even their wage bills. ...[These] loss-making public investments have left governments facing a difficult choice—to subsidize directly, to protect and so shift costs to other sectors, or to close plants and write off the losses.” (p.109)

These observations, with different severity, can be taken as indicators for the voracity effect among oil economies.

In the case of Nigeria, Lane and Tornell(1996) identify two important primitives of fiscal adjustment to the oil windfalls. First “any national government that wished to survive had to keep a balance in the allocation of federal resources across the most powerful ethnic groups ... [ whereas] much of this ... formally took the form of public investment projects ... [which] were typically a cover for kickbacks to the elites from each ethnic group and not chosen on efficiency grounds. One estimate is that 75 percent of construction costs was diverted by corruption.” (p.217)

Second, they show that “the distribution of revenues across the states was a matter of singular political importance.” (p.217)

In Gelb’s words “public capital spending accelerated rapidly from 3.6 percent of nonmining GDP in 1970 to 29.5 percent by 1976. This acceleration was so strong that it alone absorbed more than the entire increase in oil income between 1970 and 1976. The excess ... resulted in a substantial deficit.” (p.241)

As another way of assessing the extent of fiscal budget fragmentation, Easterly and Levine(1997) report that a “government-appointed commission of inquiry was unable to account for what happened to much of the 1990s government oil windfall.”

One fact that may be extracted from these observations is that windfalls



in some oil economies raise the level of expenditures by more than the size of windfalls themselves.

### **2.2.5 Size of GDP**

The adverse size effect of natural resource abundance and resource booms in some oil economies were well documented in the literature. Lane and Tornell(1996) report three country-specific examples: Venezuela, Nigeria and Trinidad and Tobago. In the first case during 1970-90, the terms of trade of Venezuela grew at an annual rate of 13.7 percent while the output per capita fell by an average of 1.4 percent per year. In Nigeria, despite enjoying the oil windfall of the 1970s and early 1980s, GDP per capita on average contracted between 1970 to 1990 by more than 4 percent annually. Trinidad and Tobago is another example of a falling of GDP in a booming economy where GDP per capita contracted at a rate of 2.75 percent per year over 1980 - 90.

Gelb(1988) estimates that in Nigeria “the average size of its nonmining economy in 1979-81 was 29 percent lower than it would have been had the country managed to sustain its preshock growth trend.” (p.81) A substantial part of Nigeria’s windfall gains “were eaten up in the reduced efficiency of its non-oil economy.”(p.254)

In the case of Venezuela, “by 1983 the nonmining Venezuelan economy was one-third smaller than it would have been had it continued to grow at the pre-1973 rate.”(p.122)

In examining eleven major Latin American economies, over the period 1960-94, Sachs and Warner(1999) also find that in four cases (Bolivia, Mexico, Peru and Venezuela) per capita GDP actually declined during and/or after the boom

period.

From these findings, one may conclude that, despite their positive direct contributions, oil booms may lower the size of GDP, leading to a negative level effect.

### **2.2.6 Some of the regularities of oil economies**

To provide a guideline for our modelling exercise in the next section, we now arrange the empirical observations of performances of oil economies in the context of some "stylized facts":

1. Oil economies may afford a higher proportion of rent-seeking relative to other economies.
2. These economies responded differently to oil booms. Except for a handful successes, for most oil economies, windfalls created a loss rather than a gain.
3. Despite their dissimilarities in size, political regimes and degree of openness, the failures performed very similarly in facing oil windfalls.
4. The pre-boom level of rent-seeking, partially explains how the economy reacts to the boom.
5. Oil abundance, coupled with the presence of powerful social groups, may cause diversion of human resources toward rent-seeking.
6. A windfall may raise the level of expenditures by more than its own size.
7. Despite their positive direct contributions, oil booms may lower the level of GDP, leading to a negative level effect.

The facts outlined here summarize some of the aspects of the performance of oil economies we are concerned about in this Chapter. Having rent-seeking at the heart of our theoretical explanation, we do not, however, claim that the picture presented here is totally attributed to that. What we observe from the poor performance of oil economies, may or may not be caused by rent-seeking.

By examining the extent of fitness of the empirical studies with the above facts, one can observe that the proposed model captures some important aspects of the reality. The reader should, however, bear in mind that the performance of the economies at hand may deviate from the stylized patterns predicted by our simple model. It obviously is not the only possible theoretical explanation for the effect of oil booms on the allocation of human resources and the level of GDP, but according to the empirical evidence, it is a plausible one.

## **2.3 The model**

Consider an economy populated by sufficiently large agents. The economy produces one homogeneous good. It also owns a pool of natural resources whose extracted flow is used abroad. The royalty on the latter can be transformed one-to-one to the former.

### **2.3.1 Public goods and government finance**

There is a government produces a public good which makes the economy more productive, engages in taxation and transfers, and collects royalties on the extraction of natural resource.

The government can convert one unit of final good into one unit of the public good. Let  $G$  be the amount produced. The government also uses  $R$  units of its

resources to provide fiscal transfers through a grant-allocation scheme. It taxes production at an exogenous rate  $\tau \in (0, 1)$ . The total tax collection is

$$T = \tau Y, \quad (2.1)$$

units of final good, where  $Y$  is the aggregate output. The government receives a royalty of  $Q$  units of final good from a foreign entity. All economic activity occurs simultaneously. In particular the public good is produced concomitantly with the final good.

The government budget is assumed to be in balance. Public revenue is  $T + Q$ . These are the funds available for production of the public good,  $G$  and the fiscal transfers,  $R$ . The balanced budget requirement implies that,

$$T + Q = G + R. \quad (2.2)$$

### 2.3.2 Labour market

There are  $L \gg 0$  identical agents each endowed with some talent. They can engage in two mutually exclusive types of activities: rent seeking and production. Let  $N = nL$  be the number of rent-seekers, where  $n$  is the (endogenous) proportion of the population engaged in rent-seeking and  $(1 - n)L$  agents engaged in production, the workers.

#### Production

Each agent engaged in production produces according to the constant returns to scale production function  $y = AlG$ , units of the goods where  $A$  is a productivity parameter,  $l$  is the labour input and  $G$  is the supply of public services. Each worker has  $l = 1$  unit of labour available which is supplied inelastically. The

aggregate product is<sup>3</sup>

$$Y = A(1 - n)LG. \quad (2.3)$$

The reward of a worker is the after tax wage, which is

$$W = (1 - \tau)AG. \quad (2.4)$$

Rent-seekers do not pay tax and only workers, by assumption, act as the *tax base*. Limited liability constraint dictates that the government cannot tax more than the revenue of workers which means  $T \leq WL(1 - n)$ . This implies  $\tau \leq 0.5$ .

### Rent-seeking

The government announces, prior to any economic activity, a granting scheme which specifies the exogenous number  $k$  of grants available, all of equal size. A rent-seeker can put in multiple applications. The total pool of money available,  $R$ , will be endogenously determined in the following section.

Lobbying is a risky activity as, typically, the number of agents engaging in lobbying,  $N$ , is greater than the number of grants available,  $k$ . It is also a costly activity. The typical rent-seeker spends  $x$  units of final good on his lobbying activity.

Grants are allocated independently. For each grant opportunity, the successful rent-seeker receives a grant of  $R/k$  and the unsuccessful one receives nothing. The probability of a rent-seeker winning the grant is proportional to his expenditure relative to aggregate outlays. That is, let  $x_i$  for  $i = 1, 2, \dots, N$  be

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<sup>3</sup>Any other production function like  $f(n).G$ , with  $f : [0, 1] \rightarrow \mathbb{R}^+$ ,  $f' < 0$  and  $f(1) = 0$ , can be taken instead. What is essential here is the linearity in  $G$  where rent-seeking intensity  $n$ , reduces the effectiveness of the public services. Since the model is static we also abstract from capital in the production function.

the outlay of  $i$ -th rent-seeker who has a probability  $x_i/X$ , of winning that grant where  $X \equiv \sum_{i=1}^N x_i$  is the aggregate outlay on lobbying. He may end up with no grant, or with  $1, 2, \dots$ , or up to  $k$  grants.<sup>4</sup>

It is clear that for  $j = 0, 1, \dots, k$  the probability of rent-seeker  $i$  ending up with  $j$  grants is the Bernoulli probability

$$P_i(j) \equiv \Pr \{j \text{ successful applications} | x_i, X\} = \binom{k}{j} \left(\frac{x_i}{X}\right)^j \left(1 - \frac{x_i}{X}\right)^{k-j}.$$

The expected value of the ‘prize’ won  $\tilde{V}_i$ , i.e. the total amount of grant money received, is (see appendix A.1)

$$\tilde{V}_i = \sum_{j=0}^k \frac{R}{k} j P_i(j) = \frac{R x_i}{X}. \quad (2.5)$$

We abstract from the problems of income distribution by assuming that all the agents live in extended families that provide insurance to their members. These families include both workers and rent-seekers and are large enough to make agents risk neutral.<sup>5</sup> We also abstract from problems of timing by assuming that all activities are conducted simultaneously. Hence, the problem of rent-seeker  $i$  is to maximize his expected reward  $V_i$  from lobbying

$$V_i = \max_{x_i} \left\{ \tilde{V}_i - x_i \right\}.$$

The  $i$ -th player chooses his outlay,  $x_i$  such as to maximize his expected payoff,  $\tilde{V}_i$ . Maximization leads to the First Order Condition of  $R(1 - x_i/X) = X$ . Summation over the number of players, by running  $i$  from 1 to  $N$ , gives the total outlays as

$$X = R \left( 1 - \frac{1}{N} \right). \quad (2.6)$$

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<sup>4</sup>See Berry(1993) for a detailed description of multi-winner rent-seeking contests.

<sup>5</sup>A winner-help-looser mechanism, e.g. à la Baik(1994), also justifies this argument.

From this, rent-dissipation, i.e.  $X/R$  for finite  $N$  is less than one indicating that rent-seeking is profitable.<sup>6</sup>

In a symmetric equilibrium, i.e. when  $x_i = x$  for  $i = 1, \dots, N$ , each player is an active rent-seeker who spends

$$x = \frac{R}{N} \left( 1 - \frac{1}{N} \right),$$

into the scheme. The symmetric reward to the rent-seeker is therefore<sup>7</sup>

$$V = \begin{cases} R/N^2 & N > 0 \\ 0 & N = 0 \end{cases}. \quad (2.7)$$

### Allocation of labour

At an interior equilibrium, agents must be indifferent between becoming a worker or a rent-seeker. The proportion  $n$  of rent-seekers will be such that

$$V = W. \quad (2.8)$$

Otherwise, the corner solutions are associated with

$$W \geq V \text{ and } n = 0, \quad (2.9)$$

$$W \leq V \text{ and } n = 1.$$

### 2.3.3 Lobbying and size of transfers

We assume that a substantial amount of rent is associated with oil revenue<sup>8</sup> where the government is the major recipient and in charge of its distribution through

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<sup>6</sup>This, in Tullock's context is called efficient rent-seeking. See Perez-Castrillo and Verdier (1992) or Nitzan(1994) for details of rent-seeking contests.

<sup>7</sup>One might instead assume that the first rent-seeker can take all he can get.

<sup>8</sup>Weitzman(1999) estimates the amount of rent associated with crude petroleum, which is chiefly distributed among few oil producers, equal to some \$170bn in 1994. In small oil exporters, Auty(2001) estimates the share of mineral rents between 20 and 30 percent of GDP.

a grant allocation scheme. Along the line of Tornell and Lane(1999) we assume that “the fiscal authority has no objection of its own” and “acts solely as the agent of powerful groups”.<sup>9</sup> This captures the fact that “fiscal policies in many countries are determined by powerful interest groups.”

The decision on the distribution of oil rents<sup>10</sup> is made by the fiscal authorities who are not rewarded for their sound economic decisions. We suppose that the grant allocation scheme is subject to lobbying activities aiming to increase the share of transfers.

Access to the resource rent in oil economies depends heavily on the pattern of public spending<sup>11</sup>. This latter is affected by cultivation of the fiscal authorities’ interests, hence lobbying is an effective way to secure access to a share in the oil rent.

Karl(1997) addresses the link between power and plenty in mining states which “own the centre of accumulation, extract or receive windfall revenues from the international arena, benefit from rents, and provide the means through which

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<sup>9</sup>One could alternatively assume that the decision making is centralized in the hands of a *semibenevolent* government, who maximizes a weighted sum of social welfare and the contributions of rent-seekers through their lobbying activities.

This approach, outlined by Grossman and Helpman(1994) and Persson and Tabellini(2000, sec. 7.3), lays out a framework for welfare analysis. It is, however, beyond the scope of our study which aims to explore the *level effect* of booms via the allocation of talent in a decentralized economy.

<sup>10</sup>By oil rents we mean the residual component of the market price net of the extraction cost where all reproducible factors are paid at market rates. This is different from *Hotelling rent* which is associated with exhaustibility and reflects arbitrage between holding the resource and interest-bearing assets.

<sup>11</sup>Tabibian(1992) shows that in the case of Iran, the large cities have been located and grown in a way to facilitate digestion of the oil revenues through fiscal expenditures.



these rents enter the economy, they become the primary object of rent-seeking behaviour.” (p.15) She refers to the specific types of organized interests and patterns of collective action which “are linked directly to the state and that benefit from oil rents. These classes and interests have strong reasons to reinforce petrolization as a means for realizing their demands.” (p.16)

Along this line, we assume that rent-seekers institute a mechanism to influence fiscal authorities for access to public transfers<sup>12</sup>. This mechanism, following Becker(1983), is introduced using the *influence function* which maps the interests of individuals into the policy outcome. The proportion of the population engaged in rent-seeking,  $n$  and the size of the royalty available,  $Q$  are two crucial primitives of the outcome of the lobbying process that produces the level of fiscal transfers<sup>13</sup>. The mechanism is defined as follows:

**Definition 1** An *influence function* denoted by  $R = R(n, Q)$  represents the reduced form of an institutional arrangement by which the rent-seekers whose relative size is  $n$  extracts fiscal transfers,  $R$  financed by oil rent,  $Q$  from the government. The function by assumption is increasing and differentiable in both  $n$  and  $Q$ . Moreover it is restricted to be nonnegative and not exceeding the oil rent, i.e.  $0 \leq R(n, Q) \leq Q$ .

Fiscal transfers is an endogenous variable determined by two primitives: the relative size of rent-seeking and the available rent. Dependence of  $R$  on  $n$  can be interpreted as an open access mechanism by which more rent-seekers claim

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<sup>12</sup> “Generally governments do not transfer the rent of resource by their own. They have to be lobbied or pressured into doing so by the expenditure of resources ( or the effort supplied ) in political activity.” (Tullock, 1967).

<sup>13</sup> The process described here though seems like an aggregate production function, but due to its *collective* nature cannot be disaggregated into the efforts that rent-seekers supply separately.

more resources. This effect is captured by Torvik(2002) through an endogenous tax rate.

Continuity of  $R$  in  $n$  is required to examine the existence and uniqueness of the equilibrium.<sup>14</sup> Furthermore one needs to ensure that a small change in the relative size of rent-seeking or a small perturbation in the level of oil rent has a relatively small effect on the level of fiscal transfers and hence combination of public expenditure. We need also differentiability of  $R$  to examine the marginal productivity of lobbying for further investigation.

The higher the relative size of rent-seekers, the more powerful and influential is their lobby and so the more effective is the political pressure for diversion of oil income toward public transfers<sup>15</sup>. Additionally, the higher  $Q$ , the bigger is the size of the cake and hence, *ceteris paribus*, the more valuable is the prize to the rent-seekers.

Non-negativity of  $R$  means that in this economy, transfers do not act as redistribution of resources from one group to another. In other words, people do not provide the source of transfers. They receive it as a gift provided by nature. Moreover we assume that oil rent is the only source of transfers in this economy.

By introducing lobbying as the main motive of transfers, we do not preclude other motivations of the fiscal authority for so doing. In this case the lower and upper bounds for  $R$  are not necessarily binding.

**Remark 1** *In case where lobbying by rent-seekers is the only cause of fiscal transfers, it is plausible to assume  $R(0, Q) = 0$  and  $R(1, Q) = Q$ . If there exist, how-*

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<sup>14</sup>Formally this means given  $Q > 0$ , for any  $L$  and  $n \in \{0, 1/L, \dots, (L-1)/L\}$ , there exists an  $M = M(L, n)$  such that  $R(n+1, Q) - R(n, Q) < M$ .

<sup>15</sup>To rationalize this effect, one can interpret total contribution of rent-seekers,  $X$  as an index of political capital that affects the policy outcome in lobbying process.

ever other rationales for fiscal transfers, e.g. reducing inequality, we can assume  $R(0, Q) \geq 0$  and/or  $R(1, Q) \leq Q$ . Unless otherwise stated, during this Chapter the latter case is assumed.

From (2.3), nothing is produced when all are rent-seekers. It does not make sense therefore to assume that a portion of public resource is allocated to productive services for producing nothing. From Eqs. (2.2) and (2.3) we obtain  $G = Q - R(1, Q)$  at  $n = 1$ . We do not, however, impose any restriction on exhaustion of oil income at full rent-seeking. What we have assumed here is the minimum constraints required to link the allocation of talent to the fiscal transfers<sup>16</sup>.

Competition among the pressure groups who demand favours determines the equilibrium level of fiscal transfers which induce these favours. The influence function is a reduced form that represents the outcome of the competition in the form of a political equilibrium, without analyzing the political system within which these activities take place. We deliberately abstract from microfoundations behind the political influence and simply assume that rent-seekers are organized and have the power to extract fiscal transfers from the government.

Two related concepts are crucial in our analysis. The *average productivity of lobbying*,  $R/Q$  indicates the portion of fiscal transfers in oil income. This shows how accessible is the fiscal budget for the successful rent-seekers and the extent to which they can siphon public resources into their own pockets.

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<sup>16</sup> According to Murphy et al. (1993), lobbying may be convex in the level of rent-seekers' efforts because political pressure involves some costs that rise less than proportionally with the level of lobbying. These include legal expenses, hiring of experts, establishment of relations with the authorities and cultivation of their interests, extraction of information, etc. We do not impose such a condition on  $R$ .

The second concept is the *marginal productivity of lobbying*,  $R_Q = \partial R / \partial Q$ .<sup>17</sup> This measures how cautious the government is to respond to the boom in an expansionary fashion. Clearly it is hard to handle oil windfalls cautiously in the face of powerful interest groups. Organized along sectorial lines, these groups generate irresistible pressure for raising subsidies. A fragmented or decentralized decision making structure reduces the ability to spend windfall gains cautiously.

The budgetary process is a convenient and effective mechanism by which rent-seekers appropriate resources from the rest of society. The *fiscal constitution* is set as follows:

- Only proportional tax on producers' income can be levied; Eq. (2.1).
- The fiscal budget has to be balanced. This is addressed in (2.2).
- The combination of fiscal expenditure and in particular the level of fiscal transfers is determined by the influence function defined above.

To avoid ambiguities, we clarify here that the level of fiscal transfers claimed by rent-seekers is what they actually get. Since we abstract from the lobbying process in detail and ignore the deadweight loss associated with redistribution, we assume that the level of appropriation of resources by rent-seekers is exactly what is enforced by influence function. Having assumed this simplification, we use the terms "claim", "transfers" and "appropriation" interchangeably where we refer to the mechanism by which rent-seekers extract oil rent from the government.

In our model, rent-seekers have the power to extract fiscal transfers from the government, but the government, in turn, does not impose tax to finance these transfers. The transfers, instead, are financed by oil rent that otherwise would

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<sup>17</sup>Subscripts refer to partial derivatives, e.g.  $R_Q = \partial R / \partial Q$ .

be allocated to public productive spending that enhances the level of income and raises the productivity of workers.

According to Lahiri and Raimoudos-Moller(2000, p.C67)'s argument, "if everyone in the economy lobbies, lobbying has no impact on the equilibrium ... one needs the existence of at least one group of individuals who are politically passive." In the current setting, it is implicitly assumed that workers are politically passive and only rent-seekers create political pressure<sup>18</sup>.

The particular modeling adopted in the influence function is somewhat *ad hoc*, but nevertheless provides a useful framework for the analysis of a realistic political process by which public decisions are made. Rent-seekers, by lobbying, increase the size of the available pie,  $R$ . On the other hand, by lowering the share of public goods in government spending  $G$ , they cut the marginal product of working and so, the producers' income. This decreases the pay in the alternative occupation and makes the rent-seeking more attractive. Hence rent-seeking amplifies itself.

## 2.4 Cost of rent-seeking

Before analyzing the model in more detail, it seems useful to highlight some of the costs of rent-seeking at this stage. The fact that the rent-seeking sector employs agents who would otherwise be productive, introduces the direct cost of rent-seeking. There is also an indirect cost. This arises by negative externality of rent-seeking on productivity of workers, caused by diverting of public expenditures from productive services. By attempting to channel government expenditures

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<sup>18</sup>In the political game designed by Becker(1983), political influence is a zero-sum game meaning that increased influence of some groups decreases the influence of others by equal amounts.

to public transfers, rent-seekers introduce another source of inefficiency in the economy.<sup>19</sup>

The total cost of rent seeking should be measured in terms of the efforts to persuade fiscal authorities to authorize the transfers by the unsuccessful as well as the successful. In an economy where entrepreneurship is not attractive and rent seeking is common and profitable, individuals learn skills that are essential for the latter. The existence of vast resource revenue in the hands of bureaucrats and the incentive for appropriation among rent seekers when combined with poor property rights and the common access problem, provide the essential requirements for transfer of resource rent to the fiscal claimants. The successful rent-seekers encourage others to join them, implying that rent seeking amplifies itself. Another side effect of rent-seeking, which is not followed here, is its effect on people's perception about the function of markets.<sup>20</sup>

#### 2.4.1 Political distribution of rents

From Eqs. (2.7) and (2.6) we have  $R = NV + X$ . This, using balance budget (2.2), results

$$Q = (G - T) + NV + X, \quad (2.10)$$

indicating that there are three channels for spending oil revenue: net transfer to the producers,  $G - T$ ; ( expected ) net transfer to the rent-seekers,  $NV$ ; and

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<sup>19</sup>Since the size of rent is endogenous in this model, the extent of rent dissipation, according to Nitzan(1994), is only a partial and in general unsatisfactory measure of the inefficiency created by rent-seeking.

<sup>20</sup>“If income distribution is viewed as the outcome of a lottery where wealthy individuals are successful (or lucky) rent seekers, whereas the poor are those precluded from or unsuccessful in rent seeking, the market mechanism is bound to be suspect.” Krueger(1974, p.302)

waste of resources by investment in rent-seeking contest,  $X$ .

The larger the size of the rent-seeking sector, the lesser is the first component and the larger are the second and third. As one considers from (2.2), the amount of subsidy to producers, i.e.  $G - T$ , declines with the extent of rent-seeking because  $d(G - T)/dn = -R_n$  which by assumption is negative. Hence although all agents receive transfers, the size of rent-seeking intensity has a distributional effect on the direction of subsidy from producers to rent-seekers.

Eq. (2.10) refers to an important task of bureaucracy in oil-based economies. In Karl(1997)'s words: "Dependence on petroleum revenues produces a distinctive type of institutional setting, the *petro-state*, which encourages the political distribution of rents. Such a state is characterized by fiscal reliance on petrodollars." (p.16)

The allocation of oil rent among interest groups justifies the bulk of bureaucracy in these economies. The distributive task by itself rationalizes the establishment of institutions for such a purpose. Besides the inefficiency which arises from these institutions, the nature of political (outside market) allocation of resources encourages pressure groups to claim fiscal transfers. The distributional task of bureaucracy is amplified during boom periods when the higher amount of oil revenue allows the government to fulfil its promises and serve national objectives.

Eq. (2.10) also states another aspect of the economy at hand. In contrast with conventional models of political competition among pressure groups, e.g. Becker(1983), here pressure groups play a positive-sum game to grab their desired share from oil rent. Workers, though, receive less due to the rent-seeking, do not provide subsidy for other party, i.e. rent-seekers. In other words source of the

prize to the winner is not transfer from the losers.

**Remark 2** *Imposing the lower and upper bounds of  $R$  on Eq. (2.2) gives  $G - Q \leq T \leq G$  meaning that producers receive more than they pay to the government. In other words both parties benefit from oil rent. The LHS is not binding of course when  $G < Q$ .*

## 2.4.2 Under-provision of public goods

The contribution of tax revenue in the financing of public productive spending,  $T/G = \tau AL(1 - n)$  falls with rent-seeking intensity,  $n$ . In other words the higher the size of rent-seeking, the lesser is the role of tax revenue in public finance and the more the economy relies on the oil endowment.

Plugging from Eq. (2.1) into (2.2), one derives the role of oil revenue in the public productive expenditures as

$$G(n, Q) = \frac{Q - R(n, Q)}{1 - \tau AL(1 - n)}. \quad (2.11)$$

The effect of  $Q$  on  $T$  can also be obtained from  $T = \tau AL(1 - n)G$ . The following lemma shows how these are related.

**Lemma 1**  $\tau AL(1 - n) < 1$  for  $n \in [0, 1]$  so long as  $R < Q$ .<sup>21</sup>

The effect of rent-seeking intensity on  $G$  and  $T$  is summarized in the following proposition.

**Proposition 1** *If  $R < Q$ , misallocation of talent causes under-provision of productive services by government, i.e.  $G_n < 0$ . The extent of diversion of public spending away from productive services toward fiscal transfers is higher, the more*

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<sup>21</sup> Missing proofs of the theorem-type statements are provided in appendix A.2.



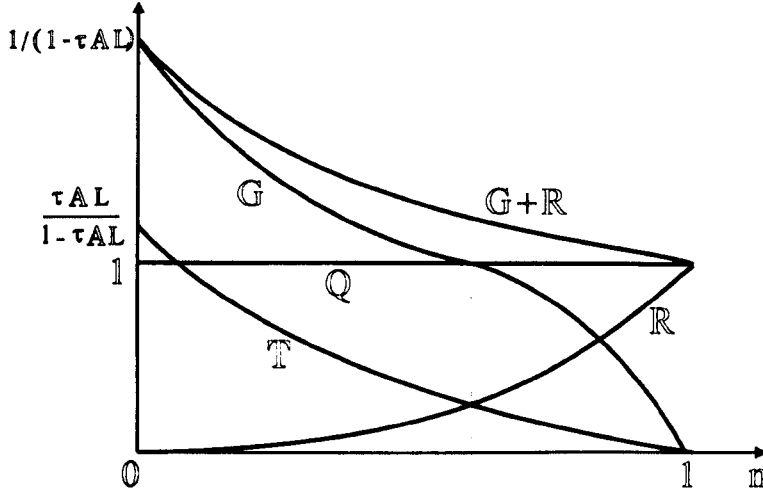


Figure 2.1: The effect of rent-seeking on fiscal variables

people engage in rent-seeking. Moreover tax income falls with rent-seeking intensity.

**Proof.** Simple differentiation gives

$$G_n = - \frac{R_n [1 - \tau AL(1 - n)] + \tau AL(Q - R)}{[1 - \tau AL(1 - n)]^2},$$

and  $T_n = \tau AL [-G + (1 - n) G_n]$  where, according to the properties of  $R$  and lemma 1, both expressions are negative. ■

A particular case of the evolution of fiscal variables as a result of change in the allocation of labour, for the case where rent-seeking is the only cause of transfers and  $R$  is convex in  $n$  is captured by figure 2.1.

Although in absolute terms, total public income,  $T + Q$  declines with the extent of rent-seeking, in relative terms the size of government

$$(T + Q)/Y = \tau + Q/Y,$$

risers with  $n$ . This means that the size of government increases with rent-seeking intensity. By taking  $Q$  and  $\tau$  as exogenous, the only endogenous part of the tax base is  $Y$  which declines with rent-seeking. One may consider an alternative route where rent-seekers influence the rate of oil extraction or tax collection. In this case, the voracity of rent-seekers outweighs their side effect on cutting tax base, raising the government expenditures along with the extent of rent-seeking, as we expect.

### 2.4.3 Output loss

In this economy,  $GDP$  has two components: the level of genuine production  $Y$ , and the production of the oil sector,  $Q$ . We assume that the latter is convertible without cost to the former. The natural resource sector contributes  $Q$  units of goods without any input requirement. This implies

$$GDP = Y + Q . \quad (2.12)$$

One may now measure the loss of output caused by rent-seeking by comparing the percentage forgone production with the case where there is no rent-seeking and workers receive their highest productivity. For this purpose we introduce

$$\frac{\Delta Y}{Y} \equiv \frac{Y(0) - Y(n)}{Y(0)} = 1 - \frac{(1 - \tau AL)}{(1 - n)^{-1} - \tau AL} \left[ \frac{1 - R(n, Q)/Q}{1 - R(0, Q)/Q} \right] .$$

This is strictly increasing in  $n$  and bounded between zero and one. It assigns to the size of rent-seeking  $n$ , the percentage of production that the economy loses relative to its potential level for two reasons. Firstly, a fraction of potential producers have no contribution in production. Secondly, rent-seekers by diverting public expenditures from productive goods, prevent producers having access to the potential productive inputs they could use if there were no rent-seeking.

## 2.5 Equilibrium

### 2.5.1 Reward structure

**Definition 2** *By reward structure we mean the pair of functions,*

$$V(n, Q) = \frac{R(n, Q)}{L^2 n^2} \quad \text{for } n > 0 \quad \text{and} \quad V(0, Q) = 0, \quad (2.13)$$

*and*

$$W(n, Q) = \frac{(1 - \tau)A}{1 - \tau AL(1 - n)} [Q - R(n, Q)] \quad , \quad (2.14)$$

*which for a given level of rent-seeking and oil rent, gives payoff to rent-seekers and producers respectively.*

Dependence of  $W$  and  $V$  on  $n$  means that what individuals receive not only depends on their choice of occupation but also on how human resources are allocated between current occupations. The following statement establishes the properties of the reward structure.

**Proposition 2** *(i)  $W$  and  $V$  are differentiable in  $n$  on  $[0, 1]$  and  $(0, 1]$  respectively and differentiable in  $Q$ ;*

*(ii)  $W$  is decreasing in  $n$ , so long as  $R < Q$ ;*

*(iii)  $W(0, Q) > V(0, Q) = 0$ .*

**Proof.** These are easily obtained from the properties of  $R$ , lemma 1, proposition 1 and definitions of  $V$  and  $W$  in (2.13) and (2.14) respectively. ■

The fact that the wage rate is decreasing in  $n$ , refers to a negative externality from rent-seeking on production via lowering the share of productive public spending. In other words, more rent-seeking in society reduces the return to production.

Two forces are associated with unproductive activities. Rent-seekers grab oil income and divert public expenditures toward fiscal transfers. This is the *external* effect. There is also an *internal* effect relating to the fighting of rent-seekers with each other over the appropriation of the grants funded by the oil income. The external effect raises the reward to rent-seekers while the internal effect lowers it. The overall effect depends on the interaction of these two. Newcomers reduce the established contestants' probability of winning. On the other hand and ignoring free-riding, they increase the rent-seekers' power to claim oil rent.

In the voracity effect papers, e.g. Tornell and Lane(1999), each rent seeker has open access to aggregate production so that rent-seeking activities increase the size of rents to be grabbed. Torvik(2002) contrarily assumes a rent sharing scheme among rent seekers, so that the more rent seekers there are, the lower is each rent seeker's expected income.

In our case, both effects are present. In the early stage of rent-seeking, the first effect dominates. When rent-seeking becomes popular however, the second effect may dominate. The external effect, which represents the open access mechanism, pulls in the direction of a less steep reward curve for rent-seekers. The internal effect, which captures the result of rent sharing between all contestants, acts in the opposite direction. For mere illustrative purposes, we suppose that  $V$  is U-shaped in  $n$ .<sup>22</sup> The reward to producers and rent-seekers as functions of  $n$  is illustrated in figure 2.2.

**Remark 3** *We assume the reward structure, for  $n \in [0, 1]$ , is common knowledge*

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<sup>22</sup>The U-shaped form allows for both the internal and external effects to remain present in our discussion. Nevertheless as long as  $V$  is differentiable in its arguments it can take a very general form.

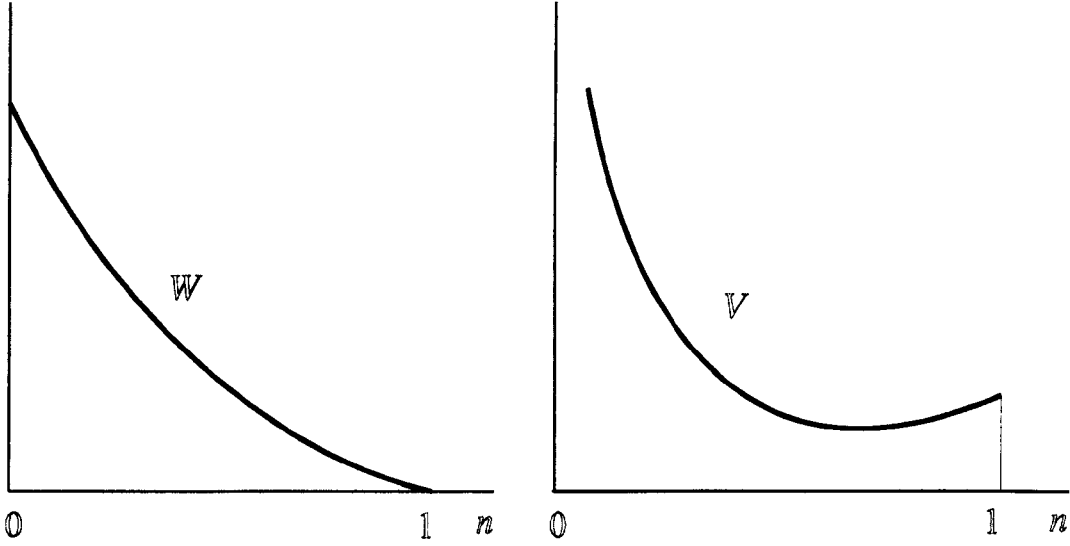


Figure 2.2: Reward to producers,  $W$  and rent-seekers,  $V$  as functions of rent-seeking intensity

*implying that everybody knows what s/he would receive, for different possible allocation of labour. Moreover, given parameters and exogenous variables, people observe the reward structure and freely choose the occupation that pays more. They do not coordinate to leave one occupation and join the other; occupational choice is an individual-based decision making.*

Considering the policy variable  $\tau$ , and the exogenous variables  $Q$ ,  $L$  and  $A$  agents by comparing  $V$  and  $W$  decide to be worker or rent-seeker. So the reward structure determines the allocation of talent. On the other hand, the allocation of human resources, through the influence function affects the reward structure itself.

### 2.5.2 The existence and uniqueness

If they are free to do so, people choose the occupation that rewards more. This gives us the allocation of labour between two existing activities according to their returns. The extent of rent-seeking further affects composition of public expenditure and hence the reward structure. A two-sided link therefore exists between the reward structure and the allocation of talent. Both are endogenous and jointly determined at equilibrium.

In this section, we define types of equilibrium and consider their existence and uniqueness.<sup>23</sup>

**Definition 3** *Equilibrium* is an allocation of labour between existing activities where, considering their payoffs, people have no incentive to change their current occupations.

Three types of equilibrium can emerge. In the good equilibrium, i.e.  $n = 0$  nobody is rent-seeker.  $n = 1$  is another corner equilibrium (called the no-activity equilibrium or full rent-seeking) where all the labour force are engaged in rent-seeking. Finally  $n \in (0, 1)$  is an interior equilibrium where there are  $nL$  rent-seekers and  $(1 - n)L$  producers.

**Lemma 2** Let  $\delta > 1/L$  be small enough, then:

- (i)  $n = 0$  is an equilibrium if  $W(n) > V(n)$  for  $0 \leq n < \delta$ .
- (ii)  $n = 1$  is an equilibrium if  $W(n) < V(n)$  for  $1 - \delta \leq n \leq 1$ .
- (iii)  $n \in (1/L, 1 - 1/L)$  is an equilibrium if

$$V(n + \delta) < W(n + \delta) < V(n) = W(n) < W(n - \delta) < V(n - \delta). \quad (2.15)$$

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<sup>23</sup>We abstract from dependence of  $W$  and  $V$  on  $Q$  whenever it is obvious and only emphasis on their dependence on  $n$ .

**Proof.** (i) If  $W(n) > V(n)$  for  $0 \leq n < \delta$ , then at  $n = 0$  by switching from production to rent-seeking, nobody will be better off and thus has no incentive to deviate. Good equilibrium is therefore an equilibrium.

(ii) If  $W(n) < V(n)$  for  $1 - \delta \leq n \leq 1$ , then the payoff of a rent-seeker in the case of switching to production reduces from  $V(1, Q)$  to  $W(1 - 1/L, Q)$ . This leaves no incentives for change of occupation at full rent-seeking.

(iii) Finally, by continuity of both  $W$  and  $V$ , when (2.15) holds,  $V$  is steeper than  $W$  at  $n = n^*$  and crosses it from below. If a producer joins rent-seeking at  $n = n^*$  he receives less because  $W(n^* - \delta) < V(n^* - \delta)$ . By the same token, if a rent-seeker at this situation leaves rent-seeking and joins productive activities s/he will be paid less because  $V(n^* + \delta) < W(n^* + \delta)$ . Both producer and rent-seeker at  $n = n^*$  have no incentive to change their occupation and the allocation of labour is therefore at equilibrium. ■

Two facts are worth noting here. First, in the case of coexistence of both activities we do not know how many interior equilibrium(s) exist.  $W$  and  $V$  are continuous in  $n$ . The former is decreasing and the latter in general is U-shaped. They can therefore intersect at several points. However, we identify conditions for the existence of such equilibria.

Secondly, our assumption on taking  $V$  and  $W$  as common knowledge, rules out the unstable roots of  $W = V$  as equilibrium because at those points attaining higher returns is possible by changing the type of activity.<sup>24</sup> When  $W$  crosses  $V$  from above, people are equally paid in both occupations. They, however, realize that by leaving rent-seeking and joining the productive activity they will be rewarded more. The labour market is not in equilibrium at these points because

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<sup>24</sup>Both Torvik(2002) and Baland and Francios(2000) ignore unstable interior equilibria. They however do not justify this restriction.

of the existence of incentive for deviation.

In similar models, e.g. Acemoglu(1995), equilibrium is defined as the points where the reward to both activities is equal. This includes both stable and unstable equilibria because so long as people are rewarded equally, they have no incentive to change their career. In our model we assume that individuals know the entire profile of  $W$  and  $V$ , meaning that the returns in both careers are commonly known for each potential value of  $n$ . This piece of *information* stimulates people to deviate from an unstable equilibrium because they know they would be better off if they left rent-seeking and joined a productive activity. For this reason, the unstable points of intersection of  $W$  and  $V$  no longer serve as equilibria in our setting.

Agents choose their professions by comparing their corresponding payoffs<sup>25</sup>. Free-entry implies that by entering into or leaving these two careers, they offset the arbitrage between  $W$  and  $V$  which implies

$$\frac{A(1-\tau)}{1-\tau AL(1-n)}(Q-R) = \frac{R}{L^2 n^2}, \quad (2.16)$$

where the LHS and RHS are wage rate and payoff to rent-seekers respectively. This implicitly defines interior equilibrium(s) that refers to the coexistence of

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<sup>25</sup>Taking morality into account, amoral agents choose their careers based purely on the corresponding payoff but moral agents for being rent-seekers and appropriating the income of the producers must incur a moral cost. The model analysed here assumes that moral cost of being rent-seeker is zero and everybody is a potential rent-seeker. Baumol(1990, p. 897-8) writes :“ If entrepreneurs are ... ingenious and creative in finding ways that add to their own wealth, power, and prestige, then it is to be expected that not all of them will be overly concerned with whether an activity that achieves these goals adds much or little to the social product or ... even ... it is an actual impediment to production.”. Also it has been assumed here that there is no social or institutional control over the diversion of talent.



productive and unproductive activities for  $0 < n < 1$ .

According to (2.16), producers join rent-seeking until the expected marginal reward of rent-seeking equals the opportunity cost of leaving the production sector. In rent-seeking contests, the number of participants are either constant or determined by free entry into the contest. In our model, there is no barrier to prevent participation in the contest, but wage rate as the opportunity cost of being unproductive provides an indicator for agents to decide whether to join rent-seeking or not.

For a net wage rate, one finds  $W(n, Q) = W(0, Q) - A(1 - \tau)[G(0, Q) - G(n, Q)]$ , which states that due to diversion of public spending to public transfer, the workers are underpaid and “diversion...acts like a tax on output.” (Hall and Jones, 1999). The extent of this implicit tax rises with rent-seeking intensity. Hence, although rent-seekers do not play an explicit predatory role, by channeling the public spending toward unproductive activity, they act as a tax burden on the production.<sup>26</sup>

In the following, we identify conditions that establish the existence of equilibria:

**Condition 1**  $W(1/L) > V(1/L)$ .

**Condition 2**  $\exists n \in (0, 1)$  such that  $W(n) > V(n)$ .

**Condition 3**  $W(n) < V(n)$  for  $1 - 1/L \leq n \leq 1$ .

**Condition 4**  $\exists n \in (0, 1)$  such that  $W(n) < V(n)$ .

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<sup>26</sup>Murphy et al. (1991) argue that “in some countries entrepreneurs ... avoid the tax from rent seekers by becoming rent seekers themselves. ”

**Proposition 3** *Existence of different types of equilibrium*

(i) If condition 1 holds, then the good equilibrium is a feasible regime. (ii) If condition 3 holds, full rent-seeking is feasible. (iii) If condition 1 does not hold but condition 2 does, or if condition 3 does not hold but condition 4 does, at least one interior equilibrium exists.

**Proof.** Apply lemma 2, bearing in mind the continuity of both  $V$  and  $W$ . ■

These conditions generate sixteen different situations. The following rules however preclude infeasible situations.

**Rule 1** *If Condition 1 holds or Condition 3 does not hold, then Condition 2 holds.*

**Rule 2** *Condition 2 and 4 cannot be violated at the same time.*

**Rule 3** *If Condition 1 does not hold or Condition 3 does, then Condition 4 holds.*

Rules 1 and 3 are based on the continuity of  $V$  and  $W$ . Rule 2 is based on logical reasoning. Having imposed these rules, the remaining feasible situations are presented in table 2.3.

The remaining six feasible situations refer to the cases where a combination of the conditions 1 to 4 hold. Figure 2.3 illustrates a situation where all three types of regimes are feasible. This captures cases (ii), (iii) and (iv) in table 2.3 where each simple case justifies the feasibility of the good equilibrium, full rent-seeking and interior equilibrium respectively.

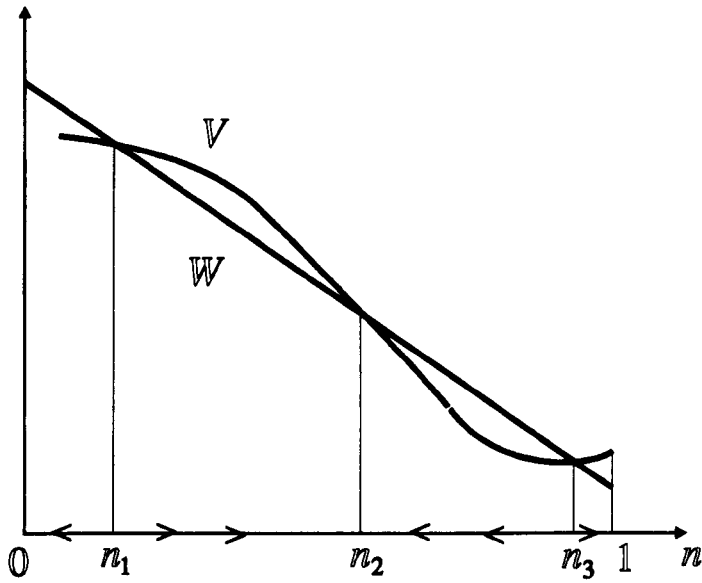


Figure 2.3: Three regimes are feasible: the good equilibrium ( $n = 0$ ), coexistence of both activities ( $n = n_2$ ) and full rent-seeking ( $n = 1$ ).

Table 2.3 Different types of equilibrium

<i>Conditions</i>				<i>Case</i>	<i>Types of equilibrium emerge</i>
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>		
×	✓	✓	✓	i	Interior Equilibrium and full rent-seeking
✓	✓	✓	✓	ii	Good Equilibrium and full rent-seeking
×	×	✓	✓	iii	Full rent-seeking
×	✓	×	✓	iv	Interior Equilibrium
✓	✓	×	✓	v	Interior Equilibrium
✓	✓	×	×	vi	Good Equilibrium

In figure 2.3,  $n = n_1$  refers to the situation where “rent-seekers have a strength in numbers” (Murphy et. al. 1993). Rent-seeking is not attractive unless a certain number of people are involved in that activity. In other words in this

situation, the society is not secure from rent-seeking unless the relative productivity of working is sufficiently high. In  $(n_1, n_2)$  and  $(n_3, 1]$ , the reward structure encourages rent-seeking and favours diversion. For  $n > n_1$  therefore the economy ends up with  $n = n_2$  or no-activity regime, i.e.  $n = 1$  where individuals are all rent-seekers.

Condition 2 or 4 justifies the occurrence of interior equilibria. In the case of the existence of such points, according to the intermediate value theorem, the number of crossing points of  $W$  and  $V$  are odd. These points are respectively stable and unstable where the latter, since the reward structure is common knowledge, do not serve as equilibrium. In figure 2.3, apart from extreme regimes of good equilibrium and full rent-seeking, there is also room for the coexistence of productive activity and rent-seeking. In particular  $n_2$  is an interior equilibrium.

When  $n = 1$ , rent-seeking is self-sustained and the economy becomes trapped in the no-activity equilibrium. One should realize that the social and political costs of pushing the economy out of this equilibrium could be high. For such a no-activity trap, our findings are consistent with what is called the *political vicious circle*, *bad equilibrium*, *underdevelopment trap* or *persistent corruption* in Krueger(1974), Murphy et al. (1993), Acemoglu(1995) and Tirole(1996) respectively.

In a no-activity regime, all believe that rent seeking is the only route to gain, and entrepreneurs devote their resources to capturing windfall rents. The only productive activity in this situation is extraction and liquidation of oil resources to finance rent-seeking activity. In predator-prey type models of rent-seeking, a no-activity equilibrium is unlikely because a minimum amount of production is required to be taken by rent-seeking. The novelty of this approach, in comparison

with full rent-seeking situations established by Murphy et al. (1993), Baland and Francois(2000) and Torvik(2002), is to link this concept to natural resource endowment. In Murphy et al. (1993) bad equilibrium exhibits a very high level of rent-seeking and extremely low living standards. In our model however we do not compare  $W(0, Q)$  and  $V(1, Q)$ .

In Torvik(2002) it may be more profitable for entrepreneurs to fully engage in rent-seeking. In this case the only source of public sector income is the natural resource, and genuine production does not occur. In our model, rent-seekers instead of appropriation of the producers' income, have common access through their political influence to the oil rent. So rent-seekers survive as long as poorly defined property rights do not prevent them from usage of oil revenue to cover their activities.

To identify the interior equilibrium(s) and the roots of condition (2.16), the following function is proved to be useful.

**Lemma 3** *Let  $\phi : [0, 1] \longrightarrow [0, 1]$  be defined as follows,*

$$\phi(n) \equiv \left[ 1 + \frac{1 - \tau AL(1 - n)}{(1 - \tau)AL^2 n^2} \right]^{-1}. \quad (2.17)$$

- (i)  $\phi$  is positive, increasing and differentiable in  $n$  for  $n \in (0, 1]$ ;
- (ii) Both  $\phi$  and  $\phi'$  tend to zero when  $n$  tends to zero;
- (iii)  $\phi(1) = \{1 + 1/[(1 - \tau)AL^2]\}^{-1} < 1$ .

We can now reduce the problem of payoffs comparison to one of comparison of the average productivity of lobbying  $R/Q$  and  $\phi$  as defined above.

**Lemma 4** *For  $n \in (0, 1]$  and  $0 \leq R < Q$ ,  $W \leq V$  if and only if  $\phi(n) \leq R(n, Q)/Q$ .*

**Proof.** By plugging from (2.14) and (2.13),  $W \leq V$  is equal to  $(1 - \tau)A(Q - R)/[1 - \tau AL(1 - n)] \leq R/L^2 n^2$  where the latter inequality can be rewritten as

$$\frac{(1 - \tau)A}{1 - \tau AL(1 - n)} Q \leq \left[ \frac{(1 - \tau)A}{1 - \tau AL(1 - n)} + L^{-2} n^{-2} \right] R.$$

This by lemma 1, when  $R > 0$ , gives  $Q/R \leq 1 + [1 - \tau AL(1 - n)] / [(1 - \tau)AL^2 n^2]$ .

For  $R = 0$  we have  $W > V$  which equals  $\phi > 0$ . ■

According to the empirical evidence, we expect that, everything being equal, there should be a positive correlation between the level of rent-seeking intensity and the degree of natural resource abundance. The following statements show that this depends entirely on the features of the lobbying process and the characteristics of the influence function. Using lemma 4, we can identify the interior equilibrium(s) by the following statement.

**Proposition 4** *Given  $Q > 0$ ,  $n \in (0, 1)$  is an interior equilibrium if:*

- i)  $n$  solves  $R(n, Q)/Q = \phi(n)$ ; and*
- ii) At a neighborhood of  $n$ ,  $\phi$  is steeper than  $R(n, Q)/Q$ , i.e. for some  $\delta > 1/L$ :  $\phi(n - \delta) < R(n - \delta, Q)/Q < \phi(n) < R(n + \delta, Q)/Q < \phi(n + \delta)$ .*

**Proof.** By definition, an interior equilibrium is a stable root of  $W = V$  where (2.15) holds. Now apply lemma 4 considering that both  $R/Q$  and  $\phi$  are increasing in  $n$ . ■

This helps to identify a sufficient condition for the case where the level of oil income does not affect the allocation of talent.

**Proposition 5** *An interior equilibrium level of rent-seeking intensity is independent from  $Q$  if  $R$  is linear in  $Q$ . In this case, oil rent rewards both activities proportionally and does not affect the choice of occupation.*

**Proof.** Applying proposition 3 to  $R = f(n).Q$  identifies the interior solutions of  $W = V$  as those points where  $f(n) = \phi(n)$ , which is independent of  $Q$ . Moreover, rewards to producers and rent-seekers in this case are respectively  $W = (1 - \tau)A[1 - f(n)]Q/[1 - \tau AL(1 - n)]$  and  $f(n)Q/L^2 n^2$  which are linear in  $Q$ . ■

### 2.5.3 Sequence of events

Based on the building blocks of the model, we can now describe how it works. The sequence of events in this static model economy is as follows:

1. Nature selects the allocation of talent between two existing activities. This determines the level of rent-seeking intensity,  $n \in [0, 1]$ .
2. Given the available level of oil rent,  $Q$  rent-seekers claim fiscal transfers equal  $R = R(n, Q)$ , and the government announces the available level of grants. This determines the combination of public expenditure.
3. Having determined the level of fiscal transfers:
  - (a) The fiscal authority sets the level of productive services accordingly:  

$$G = (Q - R)/[1 - \tau AL(1 - n)];$$
  - (b) producers produce final products  $Y = AL(1 - n)G$ ; they pay tax,  
 $T = \tau Y$  and receive their after-tax wage  $W(n, Q)$ ;
  - (c) rent-seekers receive their net payoff,  $V(n, Q)$ .
4. The allocation of labour is such that either  $V = W$  at  $n \in (0, 1)$  or  $V < W$  at  $n = 0$ , or  $V > W$  at  $n = 1$ . The economy is at equilibrium unless it is shocked. A substantial change in one of the exogenous parts of the reward

structure displaces the economy from an old equilibrium to a new one. The allocation of labour consequently changes such that the arbitrage between the two activities disappears.

## 2.6 Oil booms

Nearly all countries that rely on oil income, regardless of their political make-up, economic structure, and institutional characteristics, have encountered common basic problems. Something it seems must dwell in the very process of oil-financed development, somewhat independent of the political system, the stage of economic advancement, or the quality of economic planning and management.

It is believed that the dependence on a particular type of government revenue, shapes “the very institutions of the state, the framework for decision-making, and the decision calculus of policy makers.” (Karl 1997, p.7). This may motivate one to ask “how can a repeated pattern be explained when it occurs across countries as dissimilar in regime type, social structure, geostrategic location, culture, and size as Iran, Nigeria, Mexico, Algeria, and Venezuela? Why, in the midst of two booms, did different governments operating in distinctive contexts make choices that seem to have produced similar results?” (p.8).

Referring to *institutional sameness* in these countries that is caused by *the origin of state revenues*, Karl argues that “different sources of revenues from commodities have distinctive impacts on the scale of the state, its degree of centralization ..., the coherence of public bureaucracies, the types of organizations adopted, [and] the patterns of policy making.” (p.14). She then describes how this *petro-state*, homogenizes much of the behaviour in oil exporters.

The surprising similarity of oil-exporting states’ overall response to oil booms,



motivates us to find out the impact of oil abundance on the allocation of human resources, size and the composition of GDP. We assume that the economy stays at equilibrium when a windfall arrives. The equilibrium condition (2.15) provides an association between the relative size of rent-seeking and the level of oil income. This association is elaborated in the following subsection.

### 2.6.1 Reallocation of labour

We have elaborated so far the allocation of labour given the level of oil income. We are now interested in exploring how change in oil revenues motivates people to alter their occupations. The key link is therefore how oil booms affect the structure of incentives.

The institutional structures and preferences induced by reliance on oil revenues, in Karl(1997)'s words "reduce the range of decision-making, [and] reward some forms of behaviour more than others ... [This may] produce a situation in which one path of action becomes far more attractive or far less costly than another, and thus they can define preferences by creating overwhelming incentives for decision-makers to choose (or to avoid) a specific set of policies." (pp. 9-10).

At an interior equilibrium, both activities are equally attractive. When a boom occurs, by raising the relative payoff to one activity, it makes it more attractive than the other. This motivates people to change their occupation until the arbitrage between the two activities disappears at the post-boom equilibrium. The key concept for studying the impact of the windfalls on the allocation of labour is therefore to examine how booms affect the reward structure.

To analyze the problem more concretely, consider  $V - W$  as the bonus that the economy pays to the rent-seeking. The sign of  $\partial(V - W)/\partial Q$  determines

whether the boom encourages rent-seeking or pays more to the producers. We firstly identify a situation where booms pay two activities equally and do not motivate labour to change their occupation.

**Conclusion 1** *Booms do not change the allocation of human resources if  $R$  is linear in  $Q$ .*

**Proof.** When  $R = f(n) \cdot Q$  is linear in  $Q$ , a boom with size  $\Delta Q > 0$  changes the reward to both activities proportionally as:

$$\Delta W = \frac{A(1-\tau)[1-f(n)]}{1-\tau AL(1-n)} \cdot \Delta Q, \quad \text{and} \quad \Delta V = \frac{k \cdot f(n)}{L^2 n^2} \cdot \Delta Q.$$

In this case, if  $n$  solves  $V = W$ , it also does for  $V + \Delta V = W + \Delta W$ . ■

What we learn from the above statement is that when a certain share of resource rent goes to the fiscal transfers, the level of natural resource abundance does not have an impact on the allocation of labour.

In general when  $R$  is not linear in  $Q$ , the marginal productivity of lobbying is the key indicator that determines how a boom affects the occupational choice and which activity it promotes. The following statement separately provides conditions for the cases where producers and rent-seekers benefit from a boom.

**Lemma 5** (i)  $V_Q > 0$  if and only if  $R_Q > 0$ . (ii)  $W_Q > 0$  if and only if  $R_Q < 1$ .

We provide here the necessary and sufficient condition for the case where a boom pays one activity more than the other leading to occupational change. Similar to the case of oil endowment in proposition 4, here  $\phi$  is the reference function to identify the effect of oil booms.

**Lemma 6** Let  $n \in (0, 1]$  and  $R < Q$ .  $V_Q(n, Q) \leq W_Q(n, Q)$  if and only if  $R_Q(n, Q) \leq \phi(n)$ .

These two lemmas enable us to classify the impact of a boom on the relative payoff, on the ground of the relative size of the marginal productivity of lobbying

**Proposition 6** *The effect of booms on the reward structure at an interior equilibrium depends on the relative magnitude of marginal productivity of lobbying,  $R_Q$  and  $\phi$  at that point.*

- (i) *If  $0 < R_Q < \phi$ , then  $0 < V_Q < W_Q$ ;*
- (ii) *If  $\phi < R_Q < 1$ , then  $0 < W_Q < V_Q$ ;*
- (iii) *If  $\phi < 1 < R_Q$ , then  $W_Q < 0 < V_Q$ .*

**Proof.** Use lemma 5 and 6 and note from lemma 3 that  $\phi$  is positive and bounded from above by one. ■

This shows how, by changing the relative payoff, booms make one activity more attractive than the other. In cases (i) and (ii) booms are beneficial for both activities, but since they do not reward them equally they create incentive for change of occupation. In case (iii), by lowering the wage rate of the workers, booms make productive activity less desirable. The common feature of all three cases is that, due to the effect of booms on the labour's incentive, a pre-boom allocation of labour is no longer an equilibrium in post-boom era.

From definition of  $\phi$ , we learn that it is independent of  $Q$ . The impact of booms on the reward structure and the allocation of labour therefore depends on lobbyists' reaction to the boom and how the fiscal authority facilitates their access to the windfall. This is captured by  $R_Q$ , the marginal productivity of lobbying.

**Proposition 7** *The effect of booms on the allocation of labour can be merely categorized by comparing the marginal and average productivity of lobbying.*

- (i) If  $R_Q < R/Q < 1$ , then  $0 < V_Q < W_Q$ ;
- (ii) If  $R/Q < R_Q < 1$ , then  $0 < W_Q < V_Q$ ;
- (iii) If  $R/Q < 1 < R_Q$ , then  $W_Q < 0 < V_Q$ .

**Proof.** Suppose the economy stays at equilibrium when a boom occurs. Using proposition 4, one can therefore replace  $\phi$  in proposition 6 with  $R/Q$  at an interior equilibrium. The second part of lemma 5 is now applicable to identify whether the wage rate rises or falls with the boom. ■

According to proposition 4, interior equilibria are the points of intersection of  $\phi$  and  $R/Q$  where the former is steeper than the latter. On the other hand  $R/Q \leq R_Q$  is equivalent with  $\partial(R/Q)/\partial Q \leq 0$ . In case (i) therefore  $R/Q$  falls with  $Q$  which implies a falling of  $n$  too. In cases (ii) and (iii) however  $R/Q$  rises which is equivalent with the rising of  $n$ . In case (iii) in addition  $R_Q$  exceeds one. This according to lemma 5, results in  $W_Q < 0$ .

Now we are prepared to give a necessary and sufficient condition for the case where a resource boom induces misallocation of labour.

**Conclusion 2** *Let for  $Q > 0$ ,  $n \in (0, 1)$  is a pre-boom allocation of labour. A boom diverts labour from production to rent-seeking if and only if  $R(n, Q)/Q < R_Q(n, Q)$ .*

**Proof.** Partial differentiation from  $R/Q$  with respect to  $Q$  implies  $R/Q$  increases with  $Q \iff R_Q > R/Q$ . From proposition 4, the intersection point of  $\phi$  and  $R/Q$  is an interior equilibrium where the former is steeper than the latter. Now if  $R/Q < R_Q$ , a boom raises  $R/Q$  leaving  $\phi$  intact. This raises  $n$ .

By the same token, if a boom increases  $n$ , i.e. the point of intersection of  $\phi$  and  $R/Q$ , the only cause for so doing is the rising of the latter which is equivalent to  $R_Q > R/Q$ . ■

To show how a boom affects the relative attractiveness of one activity, let us consider the following illustrative example.

**Example 1** Suppose  $R_Q(0, Q) = 0$  and  $R_Q(1, Q) > 1$ . Moreover  $R_Q(n, Q)$  is increasing, convex and continuous in  $n$  and crosses  $\phi$  from below. There exists then a unique cut-off value of rent-seeking intensity defined by  $n^* = \arg\{R_Q(n, Q) = \phi(n)\}$  such that if the pre-boom level of rent-seeking is higher than  $n^*$ , then the boom rewards rent-seekers more and supports diversion. Otherwise it reduces the extent of misallocation of talent.

**Solution 1** Take  $R_Q$  and  $\phi$  as two functions of  $n$  on  $[0, 1]$ . For low values of  $n$ ,  $R_Q$  is flatter than  $\phi$  and therefore remains below it for some  $(0, n)$ . On the other hand  $R_Q(1, Q) > \phi(1)$  by assumption. Apply the zero theorem now on  $R_Q - \phi$  to establish the existence of  $n^* \in (0, 1)$ . The uniqueness comes from the monotonicity of both  $R_Q$  and  $\phi$  and convexity of  $R_Q$ . We have therefore  $R_Q \leq \phi$  for  $n \leq n^*$ . Now by applying lemma 6, one gets the result.

Hence for  $n > n^*$ , an oil boom encourages rent-seeking while for  $n < n^*$  it pays more to the producers and makes production more attractive. In Baland and Francois(2000), a resource boom favours rent-seeking along the whole  $(0, 1]$  interval. In our model this is the case over a shorter region, i.e.  $(n^*, 1]$  where  $n^* > 0$ . Moreover in this case, the bonus that the boom pays to rent-seekers is continuous and strictly increasing in the level of rent-seeking intensity. The case is illustrated in figure 2.4.

**Change in other exogenous parts of reward structure** Apart from  $Q$  itself, the productivity parameter  $A$ , Tax rate  $\tau$ , and size of the economy represented by the total population  $L$  constitute the exogenous aspects of the reward

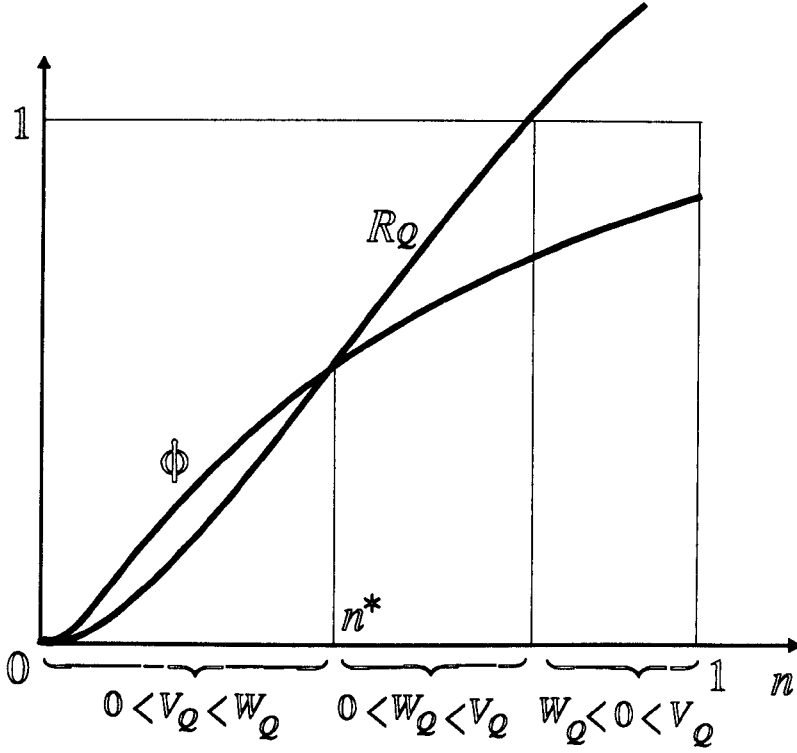


Figure 2.4: The extent of diversion that boom induces depends on the pre-boom level of rent-seeking intensity.

structure. It seems useful now to study the effect on the reward of both activities and the implied choice of occupation of any change in these parameters.

**Productivity parameter** In Eq. (2.3),  $A$  can be interpreted as the relative productivity of working to rent-seeking. A rise in  $A$ , thus, is associated with the relative rise in payoff to the producers. It raises  $W$  and leaves  $V$  intact which implies the decline of  $n$ . From (2.17), using simple algebra one can also show that  $A$ , pulls  $\phi$  upward while it does not have any effect on  $R/Q$ . Applying proposition 4 implies that  $n$  falls with  $A$ .

This accords with Torvik's result, in which an increase in productivity of

genuine production (modern sector in his words), reallocates talents in favour of productive activities. This, through a multiplier effect, increases income and welfare. The positive level effect of productivity shock is in contrast with the Tornell-Lane models, where owing to open access of multiple powerful groups to production, higher productivity may push the rate of return on investment and, thus, growth down.

**Tax rate** Raising  $\tau$  has two effects on the reward to producers while its effect on rent-seekers' income is neutral because, by assumption, they have no direct access to tax income.

By raising  $\tau$ , workers are paid less. A tax rise also provides more resources for the government to spend on productive input. The overall effect depends on the level of rent-seeking. More explicitly we have  $\partial W/\partial \tau \leq 0$  if and only if  $n \geq 1 - 1/AL$ . To put it differently, for  $n < 1 - 1/AL$ , rent-seeking intensity falls with tax rise while for  $n > 1 - 1/AL$  tax rise makes production less attractive and urges workers to join rent-seeking.

Clearly for  $AL < 1$ , the inequality does not bind and wages fall with tax rate, regardless of the size of rent-seeking. In other words, if the size of the economy in efficiency units is small, workers do not benefit from tax rise. The less expected part of the argument is that when the size of the economy, in efficiency units, is large enough, workers may gain from a tax rise.

This result can be viewed as complementary to those of Nili and Barakchian (2002) who, by taking both inequality and rent-seeking into account, find that the smaller the tax rate, the more attractive is rent-seeking for the rich. As a result one may conclude that the low tax rate in oil economies is the result of a political equilibrium which favours both poor and rich.

**Size of the economy** From (2.14) and (2.13) one learns that an increase in the size of economy denoted by  $L$ , raises the payoff to producers while it lowers the reward to rent-seekers more rapidly. As a result, by growing the economy, everything being equal, the allocation of talent changes in favour of production.

The findings on the effects of a change in exogenous parts of the reward structure on the allocation of labour are summarized in the following proposition.

**Proposition 8** *(i) A productivity shock makes production more attractive and lowers rent-seeking intensity.*

*(ii) Raising the tax rate at the low level of rent-seeking, makes production more attractive than rent-seeking, whereas when rent-seeking is common, raising the tax rate is favours the rent-seekers.*

*(iii) The larger the size of the economy, the smaller is the rent-seeking intensity.*

### **2.6.2 The voracity effect**

The concept of the voracity effect was firstly introduced by Lane and Tornell(1996) and then extended in Tornell and Lane(1998, 99) and Tornell(1999). These studies elaborate the interaction among several rent-seekers who have the power to extract fiscal transfers and claim independently a commonly accessible pool of productive asset. As a result, when the value of the asset increases e.g. by a resource boom, the powerful rent-seekers claim the windfall in a non-cooperative manner such that the size of claim grows more rapidly than the size of windfall itself.

Tornell and Lane rationalize the main failures of booming economies, like growth collapse, public debt and trade deficit on the ground of the so-called



voracity effect. They formulate how the after-effects of boom proceed when the voracious rent-seekers do not coordinate their level of appropriation.

Earlier, Gelb(1988) addressed this fact when he wrote: “some of the costs of spending boom ... are external to individual agents and are not adequately taken into account when individuals make spending and saving decision.”( p.141)

Karl(1997) reports a similar effect in the discovery of gold and silver in the Americas where the treasure exported into Spain changed the nature of international economic and political power in the sixteenth century: “in 1574, ... Philip II spent twenty-two million florins even though the government’s budget was only twelve million.”(p.36) Also: “the enormous expenditure of the state, the luxurious living of the aristocracy and the ruling class, and the widespread rentier mentality had reduced Spain to a state where people lived outside the natural order.”(p.40)

Although this study does not explain why the voracity effect operates, it rationalizes how it takes place and proceeds during booms. In order to do so and without going into the details of the cause of the voracity effect, we reproduce this concept in a simpler fashion and show how it can capture the same effect as proposed by Tornell and Lane.

**Definition 4** *The **voracity effect** refers to a situation where the size of transfers,  $R$ , grows more proportional than the size of windfall,  $Q$ . Formally this happens if  $R_Q(n, Q) > 1$ . It takes place when, besides the extra resources provided by a windfall, a boom diverts part of the productive services to the transfers. The extent of the voracity is measured by  $R_Q - 1$ .*

The voracity effect emerges in situations where transfers not only absorb windfalls, they divert some of the resources that had been allocated in the pre-

boom era to the productive services to the fiscal transfers.  $R_Q - 1$  in our model stands for what Tornell and Lane(1998) calls *propensity to suffer from voracity*.

The following statement explains the effect of a resource boom on the allocation of human resources and economic success in the presence of the voracity effect.

**Proposition 9** *When the voracity effect operates, a boom makes rent-seeking more attractive than production and motivates labour to leave the former activity and join the latter. The higher the size of a boom or the more voracious the windfall claimants, ceteris paribus, the extent of diversion of human resources into rent-seeking is more. In particular, a resource boom with a high enough magnitude induces a change of regime from the good equilibrium to the coexistence of both activities or from the latter regime to full rent-seeking.*

**Proof.** From lemma 5,  $R_Q > 1$  gives  $W_Q < 0$ . This, in conjunction with our assumption on  $R_Q$  results in  $W_Q < 0 < V_Q$  which fits case (iii) of proposition 7. As a result, a resource boom operating in the presence of the voracity effect raises the return to the rent-seeking and lowers those of producers. It displaces the interior equilibrium(s) rightward which is equivalent with misallocation of talent in favour of rent-seeking.

$W_Q = \frac{(1-\tau)A}{1-\tau AL(1-n)}(1 - R_Q)$  shows that, everything being equal, the size of falling of  $W$  owing to the boom is greater, the larger is the extent of suffering from the voracity,  $R_Q - 1$ . In addition, the amount of rise of  $V$  and fall of  $W$ ,

$$\Delta V = R_Q n^{-2} L^{-2} \cdot \Delta Q, \quad \text{and} \quad \Delta W = \frac{(1-\tau)A(1 - R_Q)}{1 - \tau AL(1 - n)} \cdot \Delta Q,$$

are linear in the magnitude of the boom,  $\Delta Q$ . ■

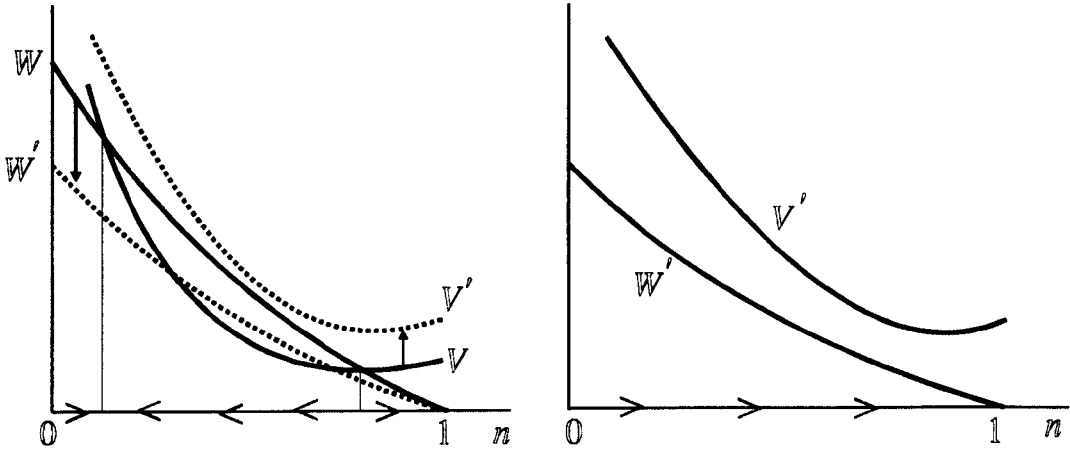


Figure 2.5: In the presence of the voracity effect, a boom may lead the changing of regime from coexistence of both activities to full rent-seeking.

Svensson(2000) addresses a striking case where an increase in government revenues may lower the provision of public goods. We show that in our setting this happens when the voracity effect operates. This provides a partial explanation for the cases in oil economies where windfalls do not lead to increased welfare.

**Conclusion 3** *Suppose  $R < Q$ . An increase in the contribution of oil rent in the government revenues reduces the provision of public productive services if and only if the voracity effect is operative.*

**Proof.** Simple differentiation gives  $G_Q = (1 - R_Q)/[1 - \tau AL(1 - n)]$ . From lemma 1,  $R < Q$  ensures that the denominator is positive. This results  $G_Q < 0 \iff R_Q > 1$ . ■

We can not fully determine in what extent the voracity effect may cause the reallocation of labour unless extra conditions are imposed on the influence function. We can however present a situation where the effect is not operative

along the whole possible range of the rent-seeking intensity.

**Proposition 10** *If lobbying is the only motive behind the fiscal transfers, the voracity effect cannot operate along the whole range of the allocation of labour between two existing activities. In particular, if marginal productivity of lobbying is increasing in rent-seeking intensity, the effect takes place only for a high level of rent-seeking.*

**Proof.** Remark 1 results  $R(0, Q) = 0$ . This implies  $V(0, Q) = 0 < W(0, Q) = (1 - \tau)AQ / (1 - \tau AL)$  for all  $Q > 0$ . Continuity of  $V$  and  $W$  implies that for each  $\varepsilon > 0$ , there exists a large enough  $L > 0$  such that

$$|V(1/L, Q) - V(0, Q)| < \varepsilon, \quad \text{and} \quad |W(1/L, Q) - W(0, Q)| < \varepsilon.$$

The first inequality implies  $V(1/L, Q) < \varepsilon$ , while the second results in  $W(0, Q) < W(1/L, Q) + \varepsilon$ . By definition,  $V(1/L, Q) = R(1/L, Q)$  which is positive but owing to the continuity of  $R$ , for large enough  $L$ , is small enough. This gives

$$V(1/L, Q) < W(0, Q) < W(1/L, Q) + \varepsilon,$$

which by condition 1, establishes that  $n = 0$  is an equilibrium no matter how large  $Q > 0$  is. As a result, the pre-boom equilibrium  $n = 0$  is an equilibrium in the post-boom era too. This shows that the voracity effect does not operate at the good equilibrium.

For the second part, suppose  $R_{Qn}(n, Q) > 0$ . The extent of suffering from the voracity,  $R_Q - 1$  hence is continuous and increasing in  $n$ . There exists therefore a unique cut-off value of rent-seeking,  $n^* = \arg\{R_Q(n, Q) = 1\}$  for which the voracity effect does not appear along  $[0, n^*]$ . ■

**Conclusion 4** *If  $R$  is separable, the voracity effect is more likely to operate only for high levels of rent-seeking intensity.*

**Proof.**  $R(n, Q) = f(n)g(Q)$  implies  $R_{Qn} = f'(n)g'(Q)$ , where both functions on the RHS are positive by definition 1. Now, apply the second part of proposition 10. ■

### 2.6.3 Income and allocation effect

By Eq.(2.12), the oil rent has a direct one-to-one contribution to  $GDP$ . The oil component rises during boom and obviously has a positive size effect. Besides this direct effect, a resource boom has two other effects. It changes the amount of available resources for productive services. It also affects the allocation of labour which indirectly affects other variables. The overall size effect of boom on  $GDP$  equals  $dGDP = dQ + dY$  where the second term on the RHS can be written as

$$dY = \frac{\partial Y}{\partial Q} \cdot dQ + \frac{\partial Y}{\partial n} \cdot dn . \quad (2.18)$$

The first and second terms on the RHS refer to the *income* and *allocation* effects of the boom respectively where the latter is obviously operative only if the boom induces a change of equilibrium. For instance, from conclusion 1, we learn that the allocation effect is neutral if  $R$  is linear in  $Q$ .

During a boom, the size of oil rents and the amount of fiscal transfers both grow where the extent of change in the latter is determined by the influence function. A boom consequently changes the available resources and directly affects the reward structure. Section 2.6.1 shows that in general, an equilibrium level of rent-seeking before a boom is no longer an equilibrium after the boom because it creates an incentive for deviation of labour. This is in effect when the

boom pays unequally to the existing activities and motivates people to change their current occupations. In this case, the allocation effect may reinforce or offset the income effect.

According to the production function (2.3), rent-seeking intensity adversely affects the level of income. Hence when a boom discourages rent-seeking, it has a positive level effect whereas in the case that the windfall makes rent-seeking more attractive, it reduces the level of production. The higher the size of a boom, the stronger is the income effect.

### Direction of effects

Because of (2.3),  $Y_n < 0$ . The allocation effect is therefore positive (respectively negative) if  $dn < 0$  (respectively  $> 0$ ). The following statement characterizes the direction of income and allocation effects.

**Proposition 11** *(i) The income effect is positive if and only if  $R_Q < 1$ . (ii) The allocation effect is negative if and only if  $R/Q < R_Q$ . (iii) When the income effect is positive, the allocation effect may be positive or negative. When the income effect is negative however, the allocation effect is also negative.*

**Proof.** (i) (2.3) and (2.11) obtain  $Y_Q = AL(1-n)(1-R_Q) / [1-\tau AL(1-n)]$ .

Lemma 1 therefore gives the result.

(ii)  $\partial(R/Q)/\partial Q = (R_Q - R/Q)/Q$ . As a result of a boom,  $R/Q$  rises (respectively falls) if  $R/Q < R_Q$  (respectively  $R/Q > R_Q$ ) while  $\phi$  remains intact. Since at an interior equilibrium,  $R/Q$  is flatter than  $\phi$ , the rise (respectively fall) of the former is equivalent with the rise (respectively fall) of equilibrium  $n$ .

(iii) When  $R_Q < 1$ ,  $R_Q$  may be less or greater than  $R/Q$ . On the other hand, since by definition  $R \leq Q$ , when  $R_Q$  exceeds one, it exceeds  $R/Q$  too. ■

## Interaction of two effects

We consider the interaction of income and allocation effects where production and rent-seeking coexist. To make it tractable, we confine our analysis to the case (iii) in proposition 3 where  $W$  and  $V$  cross at  $n_1$  and  $n_2$  ( $0 < n_1 < n_2 < 1$ ) and  $n_1$  is an interior equilibrium. Moreover we assume that the economy stays at  $n_1$  when it receives a windfall.

Considering whether  $n^* = \arg\{R_Q(n, Q) = \phi(n)\}$  is on the right or left of the interior equilibrium, two cases are distinguished.

**Case 1**  $n_1 < n^* < 1$

**Case 2**  $0 < n^* < n_1$

In case 1,  $0 < V_Q(n_1) < W_Q(n_1)$ , meaning that the boom rewards both producers and rent-seekers but pays the former group more. As a result, after a boom,  $n_1$  is no longer an equilibrium. The boom stimulates rent-seekers to join production until the incentive for deviation disappears at  $n'_1 \in (0, n_1)$ . This is depicted in the upper panel of figure 2.6.

In case 2, we have  $V_Q(n_1) > W_Q(n_1)$  and the boom favours rent-seekers more. A boom therefore induces diversion from production to rent-seeking implying that the allocation of human resources ends up with a new equilibrium at  $n'_1 \in (n_1, 1)$ . In this case, the allocation effect counteracts the income effect. The impact of a boom on the equilibrium level of the rewards is ambiguous in this case and depends on whether the new equilibrium level of returns to both activities (at  $n'_1$ ) is higher or lower than its pre-boom level (at  $n_1$ ). In the latter case, the adverse level effect of rent-seeking, generated by producers who have switched to unproductive activity, offsets the income effect of the boom. The occurrence

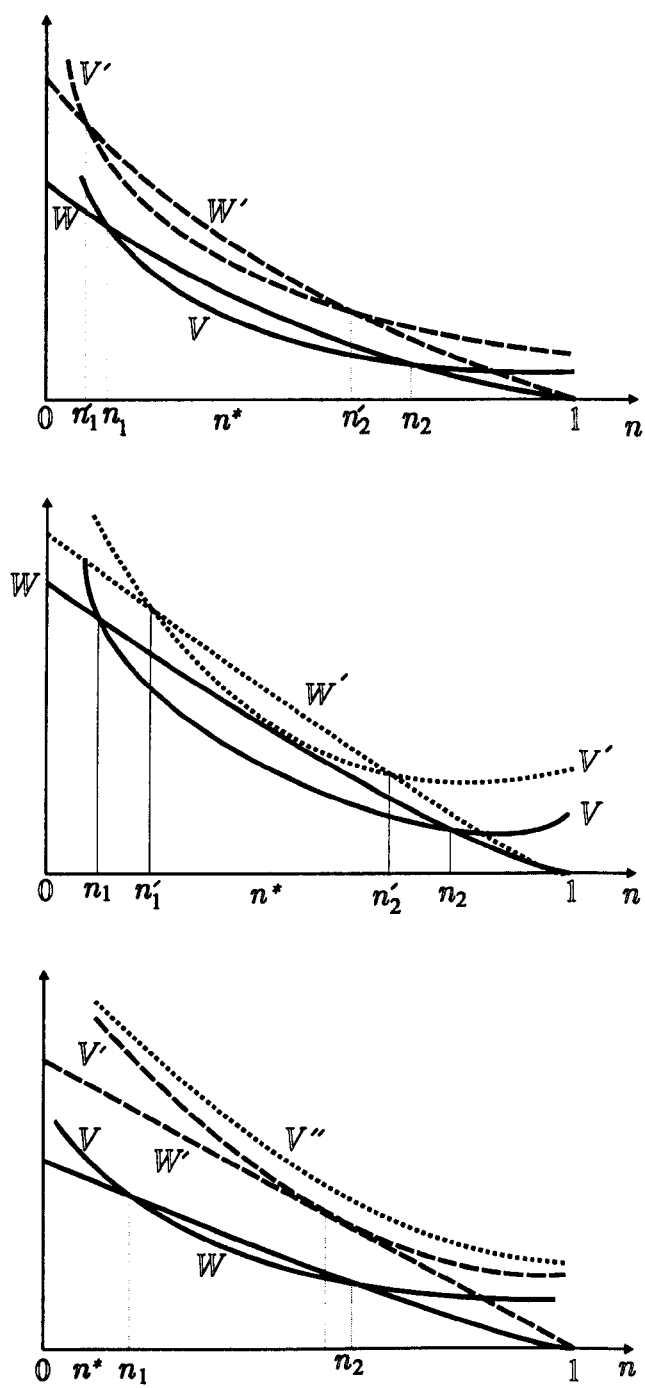


Figure 2.6: Interaction of the income and allocation effects



of this situation depends on the pre-boom equilibrium that the economy stays at ( $n_1$ ), the magnitude of boom ( $\Delta Q$ ) and the extent of change implied by rent-seeking on the reward structure ( $W_n$  and  $V_n$ ). This is illustrated in the middle panel of figure 2.6.

Case 2 also reveals a possibility for changing of regime induced by an oil boom. Since booms reward rent-seeking relatively more, the points of intersection of  $W$  and  $V$  in this case become closer after booms. The higher the magnitude of booms, *ceteris paribus*, the closer are  $n'_1$  and  $n'_2$ . Given the parameters of the model, there exists a threshold level of oil rent, say  $\overline{Q}$ , for which  $n'_1$  and  $n'_2$  coincide and  $V$  touches  $W$ . For  $Q > \overline{Q}$  therefore, there is no room for interior equilibria and only the no-activity equilibrium is feasible. In this case, the economy switches from the coexistence of both activities in the pre-boom period to the no-activity regime in the post-boom era. This is illustrated in the lower panel of figure 2.6.

In this case the misallocation of talent induced by booms totally offsets their income effects such that, overall, booms favour diversion of human resources. Moreover a temporary shock in this case has a permanent effect and the economy stays at a no-activity equilibrium even when the level of oil income returns back to its pre-boom level.

The above analysis refers to a situation where an oil boom, which is naturally expected to bring about a boost in income and economic prosperity, induces a major change in the allocation of human resources in favour of rent-seeking and ironically results in a sharp decline in the standard of living. We develop this possibility further in the next section.

#### 2.6.4 Level effect

From section 2.4.3 we learn that  $GDP$  has two components,  $Y$  and  $Q$  that refer to the production and endowment parts of the economy respectively. According to (2.3), the first component decreases in rent-seeking intensity as both the contribution of labour and size of public productive services fall with  $n$ . The size of the endowment economy  $Q$ , on the other hand is exogenous.

A change in the magnitude of the resource rent, by a resource boom, increases the size of the endowment economy and consequently  $GDP$ , one-to-one. It also affects the allocation of labour between existing activities which consequently affects the level of fiscal transfers and the public production services,  $R$  and  $G$  respectively.

The task of this subsection is to identify whether the effect of booms on the size of production economy amplifies or mitigates their direct contribution on  $GDP$ . In the former case, a boom initiates a positive multiplier effect on the income. In the latter case, the question is whether the adverse effect of a boom outweighs its positive income effect.

In contrary to Torvik(2002), here  $Y+Q$  does not increase by the same amount as an increase in  $Q$ . There is an indirect contribution from the natural resource sector to the production economy acting through rent-seeking that may operate in the same or opposite direction relative to its direct contribution.

One may argue that, since the windfall provides extra resources and hence options to the booming economy, it is difficult to make the case where it can have a negative overall value. The country, in this view, can follow, after all, exactly the same policies as it would have done in the absence of the windfall and give the windfall away, e.g. as foreign aid. This argument ignores the changes that

the windfall introduces in the domestic institutions of the host economy and the after-effects of the boom that last after the boom.

The following statement gives the necessary and sufficient condition for the adverse level effect of booms. It firstly expresses the case where a resource boom pushes the level of genuine production down. It then examines the overall effect of booms. This latter part considers the case where natural resource abundance lowers income.

**Proposition 12** *Let  $R < Q$ . (i) A resource boom shrinks the production economy if and only if either the voracity effect operates or the allocation effect dominates the income effect. More formally*

$$dY < 0 \iff (1 - R_Q)dQ < \left( \frac{G}{1 - n} + R_n \right) dn.$$

(ii) *The boom lowers GDP if and only if*

$$\left[ (1 - R/Q) \frac{Q}{Y} + (1 - R_Q) \right] dQ < \left( \frac{G}{1 - n} + R_n \right) dn. \quad (2.19)$$

**Proof.** Plugging from (2.3) in (2.18) and differentiating, obtains

$$\begin{aligned} dY = & \frac{AL}{1 - \tau AL(1 - n)} \{ (1 - n)(1 - R_Q) \cdot dQ - \\ & \left[ \frac{Q - R}{1 - \tau AL(1 - n)} + (1 - n)R_n \right] \cdot dn \}, \end{aligned} \quad (2.20)$$

which using lemma 1 and Eq. (2.11), results

$$dY \geq 0 \iff (1 - n)(1 - R_Q) \cdot dQ \geq [G + (1 - n)R_n] \cdot dn.$$

This concludes the first part.

For the second part, apply the above result into  $dGDP = dY + dQ$  which,

using lemma 1, results in:

$$dGDP \lesseqgtr 0 \iff [1/AL + (1 - n)(1 - R_Q - \tau)] dQ \lesseqgtr [G + (1 - n)R_n] dn.$$

By dividing both sides by  $1 - n$  and applying (2.3) and (2.1), the expression within the first bracket becomes  $(G - T)/Y + (1 - R_Q)$ . Now apply (2.2). ■

This proposition can offer a theoretical explanation for Gelb(1988, p.4)'s puzzling question: "Is it in fact possible for a country receiving a large windfall gain to end up less well-off than it might have been without it?"

Clearly it is expected that when both income and allocation effects, due to the voracity effect, are negative or even in the absence of the voracity effect, when the (negative) allocation effect is strong enough to outweigh the (positive) income effect, a boom can shrink  $Y$  and lower the genuine production component of GDP. What is less expected is the case where the adverse effect of a boom dominates its positive direct effect leading to an overall negative level effect. The following statement presents a sufficient condition for such a case, identifying the factors that generate the negative level effect of a boom.

**Conclusion 5** *Let  $R < Q$ . A boom reduces GDP if the voracity effect is operative and the propensity to suffer from the voracity, exceeds the size of the subsidy to the producers in units of the final good. More formally,*

$$\frac{G - T}{Y} < R_Q - 1 \implies \frac{dGDP}{dQ} < 0. \quad (2.21)$$

**Proof.** The LHS of inequality (2.19) represents the appropriately scaled measure of the sum of the direct effect of the boom on GDP and its income effect, whereas the RHS represents the allocation effect in the same scale. Obviously, when  $dn < 0$ , the RHS is negative and the inequality does not hold. In the

case where  $dn > 0$  however, the result depends on which effect dominates. One sufficient condition for falling of  $GDP$ , is the case where the LHS of (2.19) is negative. This, taking into account  $R \leq Q$ , gives the result. ■

The above result refers to a situation where the wealth of a country makes it poor<sup>27</sup>. It shows the case where the (negative) income effect of boom sufficiently compensates its direct (positive) effect, negating the overall windfall value for booming economies<sup>28</sup>.

Restating the sufficient condition for the negative size effect of boom in (2.21), as

$$\frac{Q}{Y} < \frac{R_Q - 1}{1 - R/Q}, \quad (2.22)$$

one notes that three factors accounts for this result. First, the lower the extent of resource intensity,  $Q/Y$  the higher is the relative weight of the production economy that falls with rent-seeking and therefore the likelier is the adverse effect of a boom to dominate. Second, the higher the propensity to suffer from voracity  $R_Q - 1$ , or the less cautious is windfall spending, the greater is the (negative) indirect effect of the boom. Finally, the higher the extent of diversion of oil income toward fiscal transfers, or the closer  $R$  is to  $Q$ , the more likely the adverse effect of the boom is to appear.

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<sup>27</sup>In simulation of a general equilibrium model incorporating the common features of oil economies, Gelb(1988, ch.8) concludes that for a wide range of parameters, the cost of windfalls generated by the severity of subsequent recession and stagnation, is more than its benefits. This happens because downside macroeconomic costs are potentially far larger than upside absorption benefits. By ignoring the cost of subsequent recession, we pursue a different line of reasoning.

<sup>28</sup>Even considering the benefits of windfalls for booming economies, Gelb(1988) believes that "the decade of the oil windfalls has involved the global economy in a massive, negative-sum game."(p.143)

Conversely, countries with a substantially large resource sector, or those where access to the fiscal budget is restricted are less likely to experience the negative size effect of resource boom.

**Equilibrium switching** Another possibility for lowering the level of GDP by booms is the case of equilibrium switching. Formally, this happens when, owing to a change in the exogenous part of the reward structure, the allocation of talent experiences a discontinuity. Given the exogenous variables and parameters of the model, rent-seeking intensity  $n$  by (2.16) is a function of oil rent  $Q$  which is implicitly defined by  $R(n, Q)/Q = \phi(n)$ . Equilibrium switching thus refers to the points of discontinuity of  $n = n(Q)$ .

Since occupational choice, by assumption, is an individual-based decision, equilibrium switching happens in the case where a change in the level of oil income makes a feasible regime infeasible. The most interesting case occurs when, due to a change of regime, the economy switches to full rent-seeking. In this case the *production economy* collapses but the *endowment-based economy* survives.

An equilibrium switching induced by a boom has the property that a temporary shock initiates a situation where the after-effect of the boom lasts even when the resource rent has returned to its pre-boom level.

When the allocation effect is negative, at the points of discontinuity of  $n(Q)$ , there exists a possibility for a resource boom with a small magnitude to induce a substantially large diversion of the human resources from production to rent-seeking. In this case the extent of fall in genuine production  $Y$ , may exceed the direct contribution of the boom, such that overall, the boom lowers total output.

**Conclusion 6** *In the case of equilibrium switching, a resource boom has a negative level effect if the extent of change in rent seeking intensity relative to the resource rent is sufficiently large such that*

$$[(G - T) / Y + (1 - R_Q)] dQ < [G / (1 - n) + R_n] dn. \quad (2.23)$$

**Proof.** Consider the case where  $dn < 0$ , but inequality (2.22) does not necessarily hold. Suppose  $n(Q)$  has a discontinuity from the right at  $Q = Q_0$ . This means  $\forall \epsilon > 0$ , there exists an  $M > 0$  such that  $n(Q) - n(Q_0) > M$  for  $Q - Q_0 < \epsilon$ . Now let  $Q$  rises from  $Q_0$  to  $Q_1 \in (Q_0, Q_0 + \epsilon)$ . This lowers the LHS of (2.23) to  $[(G - T) / Y + (1 - R_Q)] \epsilon$  while the RHS is bounded below by  $[G / (1 - n) + R_n] M$ . By choosing  $\epsilon$  small enough, the proof is complete. ■

### 2.6.5 Classification of the boom effects

To summarize the impact of a resource boom on the allocation of labour, size and composition of GDP, we identify three mutually exclusive cases.

**Case 3**  $R_Q < R/Q < 1$ ;

**Case 4**  $R/Q < R_Q < 1$ ;

**Case 5**  $R/Q < 1 < R_Q$ .

The impact of a boom on the economy can be summarized as follows.

**Conclusion 7** *(i) In case 3, a resource boom is gainful. It increases returns to both productive and unproductive activities, but raises the relative attractiveness of the former. For this reason a boom improves the allocation of talent in favour of productive activities and reduces the extent of diversion.*

*A boom in this case also promotes provision of public productive services. It therefore raises genuine production for two reasons and expands GDP more than proportionally. However the effect of booms on natural resource intensity is not clear.*

*(ii) In case 4, a resource boom induces both gains and losses. It promotes returns to productive activities and rent-seeking, but makes the latter more attractive. As a result, a boom in this case motivates the producers to become rent-seekers.*

*By increasing the amount of public services, a boom, on the other hand, boosts production in this case. The net effect on production depends on the interaction of the negative allocation effect and the positive income effect. The overall size effect of the boom and its impact on resource intensity is not clear in this case.*

*(iii) In case 5, a boom is likely to be counterproductive. It raises the return to rent-seekers and lowers the pay to the producers. It therefore affects incentives in favour of rent-seeking and supports diversion of human resources. A boom in this case lowers provision of public productive services. As a result, it shrinks the genuine production sector for two reasons.*

*The overall level effect depends on the size of the negative effect of the boom on the production sector and its positive direct effect. By reducing the size of the production economy and increasing the size of the endowment economy, a resource boom in this case changes the composition of GDP and raises the natural resource intensity.*

**Proof.** In case 3,  $R_Q < 1$  implies  $G_Q > 0$  meaning that the boom provides more resources for productive services and has a positive income effect. Moreover, since  $R_Q < R/Q$  the boom also lowers  $n$ . These two imply  $dY > 0$  which results



in  $dGDP/dQ > 1$ . The impact of the boom on resource intensity and thus composition of GDP depends on whether  $dY$  exceeds  $dQ$  or not.

In case 4, the income effect is still positive, but since  $R_Q$  exceeds  $R/Q$ , the boom increases  $n$ . Impact of the boom on the production economy and GDP thus is not clear.

In case 5,  $R_Q$  exceeds one, implying that both income and allocation effects are negative. A boom in this case therefore lowers  $Y$  and raises the resource intensity  $Q/Y$ . The overall size effect of the boom, however, depends on whether a direct contribution of the boom, i.e.  $dQ > 0$  compensates the shortage of genuine production,  $dY < 0$  or not. ■

Our findings on the boom effects are illustrated in table 2.4.

Table 2.4 Summary of boom effects

CASE	Income effect $Y_Q \cdot dQ$	Allocation effect $Y_n \cdot dn$	Effect on production $dY$	Welfare effect $dGDP$
$R_Q < R/Q < 1$	+	+	+	+
$R/Q < R_Q < 1$	+	−	?	?
$R/Q < 1 < R_Q$	−	−	−	?

In contrast to our findings, Grossman and Mendoza(2001) argue that “although a large resource endowment can cause people to allocate more time and effort to appropriative conflict, in a steady state the direct positive effect of abundance on consumption would outweigh this indirect negative effect.” One reason for this result is that the Grossman-Mendoza model does not address the voracity effect and hence only covers cases where the income effect of a boom is positive. Our treatment however shows that even in case 4, the negative allocation effect

of a boom may exceed its positive income effect.

An illustrative example is useful in this stage to show how each case proceeds. For simplicity and tractability we consider the case where the influence function is separable.

**Example 2** Let  $R(n, Q) = f(n) \cdot g(Q)$  be separable where  $g(Q) = aQ^3 - bQ^2 + cQ$  is a cubic polynomial with  $a, b, c > 0$ . Moreover  $f$  is such that at some level of  $Q > 0$ , all three types of equilibrium are feasible. For  $g$  we have  $g'(Q) \leq g(Q)/Q \iff Q \leq b/2a$ . Moreover for  $Q_1 \equiv \left[ b + (b^2 - 3ac + 3a)^{1/2} \right] / 3a$  and  $Q_2 \equiv \left[ b + (b^2 - 4ac + 4a)^{1/2} \right] / 2a$  with  $b/2a < Q_1 < Q_2$  we have  $g'(Q_1) = 1$  and  $g(Q_2)/Q_2 = 1$ . Since  $f(n) > 0$  is order preserving, for  $Q \in (0, Q_2)$  we have:

- (i)  $R_Q < R/Q < 1$  for  $0 < Q < b/2a$ ;
- (ii)  $R/Q < R_Q < 1$  for  $b/2a < Q < Q_1$ ;
- (iii)  $R/Q < 1 < R_Q$  for  $Q_1 < Q < Q_2$ .

In other words, as  $Q$  varies from 0 to  $Q_2$ , the economy moves from case 3 to case 4 and then to case 5. The boom effects therefore depend on the pre-boom level of oil rent  $Q$ .

For  $0 < Q < b/2a$ ,  $R/Q$  falls with  $Q$  leading to fall of  $n$  as well. The boom makes production more attractive and motivates labour to shift from unproductive to productive activities. Moreover since  $R_Q$  is limited from above, the economy is cautious in windfall spending. In this range, the boom has a multiplier effect and raise GDP more rapidly.

For  $b/2a < Q < Q_1$ ,  $R/Q$  rises with  $Q$ , raising  $n$  as well. The diversion of the labour force induced by the boom reduces the scope of its positive income effect. As a result the boom has a positive but less than proportional level effect.

*It changes the composition of GDP away from a production economy towards an endowment one.*

*Finally for  $Q_1 < Q < Q_2$ , both  $R/Q$  and  $n$  rise with  $Q$ . A boom in this case shrinks the genuine production and transforms the economy from production toward an endowment economy. In the case of the discontinuity of  $n(Q)$ , the boom might have an adverse level effect too.*

## 2.7 Conclusion

This Chapter elaborates the effect of rent-seeking on misallocation of talent when it competes, in employment of human resources, with productive activities. We tailor our model to specific aspects of rent-seeking in oil economies where the size of oil rent is substantially high and the government is in charge of the allocation of rents. Rent-seeking, coupled with lobbying, bridges the allocation of labour to the fiscal policy and links the latter to the productivity of working.

Our major findings are summarized as follows:

1. Reward to productive activities decreases in the level of rent-seeking. This is caused by a negative externality from rent-seeking on production.
2. Rent-seeking has a negative level effect for two reasons. Some potential producers choose rent-seeking rather than production. Moreover, by lobbying, rent-seekers divert public expenditures toward transfers and away from productive services.
3. Allocation of human resources between production and rent-seeking and the reward to both activities are jointly determined at equilibrium, where there is no incentive for change of occupation.

4. There is scope for multiplicity of equilibria including interior and corner solutions. This may account for divergent performances across oil economies. Multiplicity accommodates the performance of successful economies as well as their failures in a unified framework.
5. The good equilibrium, in which there is no rent-seeking, is not feasible unless the productivity of working is sufficiently high relative to that of rent-seeking.
6. Except in the good equilibrium, producers are always underpaid. The extent of this depends on the level of rent-seeking that acts as an extra tax on income.
7. In full rent-seeking, the extraction of oil reserves is the only economic activity and everybody is a rent-seeker. Rent-seeking, in this regime is self-sustained.
8. Apart from direct predation on producers, rent-seeking can take the form of making a claim on oil rents and influencing the fiscal authorities to divert productive spending to public transfer.
9. Other types of equilibrium than good equilibrium, emerge because people cannot coordinate the abandoning of rent-seeking. Agents take the reward structure as given and individually choose the occupation that pays more.
10. Resource booms, in general, reward both activities unequally and motivate people to change their occupation. Whether booms favour productive or unproductive activities depends on the characteristics of the lobbying activity and political influence. In the case where booms pay more to the

rent-seekers, it is possible that the net level effect of booms is negative.

11. Under plausible assumptions, an oil boom can cause a change in the regime from coexistence of both activities to full rent-seeking. In this case, temporary shock has permanent effects and the after effects of booms persist even when the oil rent returns to its pre-boom level.
12. The voracity effect is operative when the marginal productivity of lobbying exceeds one. In this case, a boom favours misallocation of talent and may lead the economy towards full rent-seeking.

Our analysis here leaves some questions unanswered. The model does not explain why the fiscal authorities are influenced by rent-seekers but not by producers. It does explain that the power for extraction of fiscal transfers comes from being organized. It does not elaborate, however, why rent-seekers get politically organized and producers do not.

We treat, in this approach, lobbying as a ‘black box’. In other words, our explanation of lobbying and political influence lacks a precise model of the process whereby rent-seekers get politically organized and producers do not. The ‘fiscal authority’ and the process of policy choice is also a black box. Lacking a structural model to drive the policy outcome on the ground of agents’ preferences, we cannot answer these questions properly. We believe however that our simple approach, by abstracting from an active fiscal policy, succeeds in shedding light on some of the aspects of the economies we are concerned about.

Our treatment of the influence function is mostly taken from Becker’s major work where competition among political pressure groups provides a positive explanation of political choices. This has been evaluated by Laffont(2001, pp.6-10)

as a “demand-determined political equilibrium” which leaves “the supply side of political favors” unmodeled. Applying the “fundamental modeling of information” to “open the black box of the supply side”, may therefore be fruitful in unveiling some aspects of the booming economies that are not covered in this study.

In our model, we implicitly assume that occupational choice is reversible. By this assumption we deliberately abstract from the dynamics of the economy when it converges to the equilibrium. Acemoglu(1995) in his dynamic setting, assumes that occupational choice is not reversible. By taking a constant and common rate of birth and death, he makes room for skill accumulation where the current holders of the inferior profession gradually die and the newcomers choose an activity that pays more. This defines a constant rate of adjustment for the economy and convergence toward its equilibrium.

The underlying structure, however, can support some of the dynamic features of diversion of human resources and their implied costs. Following the seminal work of Tullock(1967), income transfers lead people to employ resources in attempting to obtain such transfers. This encourages individuals to learn and accumulate skills that are essential for rent-seeking. In Acemoglu(1995)’s words “choosing a career is a commitment to hold an asset for the rest of one’s life.” The environment where individuals accumulate skills, according to Hall and Jones(1997), is an important part of the explanation of the large differences in economic levels across countries.

Hence, when the reward structure favours diversion, the types of skills that individuals accumulate would be those that maximize their productivity in rent-seeking, instead of skills that would increase the productive capacity of the econ-

omy. In these situations “investment in capital, skills and new ideas is reduced by the threat of diversion. Moreover, some of the investment that does take place is devoted to increasing the effectiveness of diversion instead of the effectiveness of production.” (Hall and Jones 1997, p.175) This is why some countries invest so much less than others.

To make the social climate favourable for production, two elements are proposed in the literature. Firstly, people should be *taught* about the social cost of rent-seeking (Hall and Jones, 1999); society should be armed with an *anti-rent-seeking ideology* (Murphy et al., 1993) to facilitate the required *cultural change* (Baumol, 1990).

The second element is *changing the rules to offset undesired institutional influences* (Baumol, 1990); *raising productivity* of production (Murphy et al., 1993) and *protecting* it by a *credible threat of punishment* (Hall and Jones, 1999).

In line with Acemoglu (1995), our study shows that when the reward structure and the allocation of talent are endogenous and jointly determined, a policy addressing the diversion of human resources is not easy to implement. For an economy which gets trapped in an equilibrium with acute rent-seeking, the policy should aim to influence the reward structure to make rent-seeking less attractive, raise the productivity of production and protect innovators from predation.

## Chapter 3

# Transitional Dynamics of the Lucas-type Growth Models

*The steady state is not a bad place for the theory of growth to start,  
but may be a dangerous place for it to end.*

Solow(1970, p.7)

### 3.1 Introduction

Multi-sector endogenous growth models have received great attention in recent years, though, due to intractability, the dynamics of these models are not yet well understood. Because they initially proved to be intractable, “much of the analysis was restricted to balanced-growth paths ..., or to analyzing transitional dynamics using numerical simulation methods.” Turnovsky(2000, p.502)

It is quite common to abstract from transitional dynamics, by assuming that the steady state growth path can be reached instantaneously; Barro and Sala-i-Martin(1995, ch. 6 and 7) and Aghion and Howitt(1998, ch.5). This, from an



analytical standpoint, provides a great deal of convenience. It makes the analysis so tractable and in some situations approximates the reality quite well. Moreover, by taking advantage of this simplification, “problems such as time consistency of the optimal policy do not arise.” Turnovsky(p.487)

The convenience, however, comes at some costs. The steady state analysis describes a situation which is never attainable in finite time. Quantitative analysis shows that the actual economies move approximately 4% of their remaining distance toward their steady state each year; Romer(2001, sec. 1.5). They spend most of their time, therefore, on the transition rather than their steady state paths. The argument that all the variables in the economy grow at their long-run growth rates, may therefore be misleading in some situations. Furthermore, this analysis does not allow for the model to accommodate policy changes and other shocks that are continually occurring. It is therefore important to extend the growth models, where necessary, to allow for the coexistence of both steady state and transitional paths.

Most of the studies that do not restrict their analysis to steady state, either tackle the transitional dynamics by numerical methods; Mulligan and Sala-i-Martin(1993), or approximate the original dynamics by the behaviour of the linearized system around the steady state; Bond et al.(1996). The current study, instead, is not restricted to the vicinity of the steady state. Our treatment is also based merely on analytical methods.

We examine the transitional dynamics of a two-sector endogenous growth model in the tradition of Lucas(1988). We focus on the dynamics of consumption, physical capital and output growth. Although investigating the transitional dynamics along the whole off-balanced path is not in general tractable, we show

that the Lucas assumption, by which only one type of capital is allocated across both sectors, allows us to fully explore the dynamics of consumption and physical capital along the transition path. The dynamics of output growth, however is only identified around the steady state, unless one allows for a very low degree of consumption smoothing, which enables us to specify the global dynamics of output growth too.

We argue that the exposition outlined in this Chapter is fruitful both from theoretical and applied standpoints. We contrast our findings with those of Caballé and Santos(1993) and show that the dynamics of consumption and physical capital in the Lucas model differs from that of Ramsey. We also demonstrate the novelty of this study with respect to those of Mulligan and Sala-i-Martin(1993) and Barro and Sala-i-Martin(1995, sec. 5.2.2).

According to our findings, in a Lucas economy, with a high enough ratio of physical to human capital, both consumption and physical capital fall along their off-balanced paths. They decline for a finite period and then rise on their transition, toward the steady state. When the output-capital ratio is far short of its steady state and consumption smoothing is strong enough, four episodes occur during transition. First, both consumption and physical capital fall. Next consumption rises but physical capital still decumulates. In the third phase, both variables rise with different rates and finally, at the steady state they grow at a common and constant rate. The time profile of consumption and physical capital is therefore U-shaped, where the minimum of physical capital lags behind those of consumption. A numerical exercise suggests that the *falling period* is not very far from the vicinity of the steady state.

The second stage, in which consumption and physical capital move in opposite

directions, distinguishes the Lucas-type models from that of Ramsey. In the *exogenous growth regime* where the Lucas model reproduces the dynamics of the Ramsey, this stage disappears and consumption and physical capital move in a parallel fashion along their U-shaped paths. In the *paradoxical regime*, in which agents are very impatient, however, in the second stage, the fall of consumption coincides with a rise in physical capital while the other stages are repeated in the same manner.

Our study also accounts for two different patterns for the local dynamics of output growth in the locality of the steady state. Besides a symmetric U-shaped form, identified by Barro and Sala-i-Martin(1995, sec.5.2), we introduce another pattern where output growth is increasing in the output-capital ratio. The latter pattern, in the exogenous and paradoxical regimes, is valid along the whole transition path.

In the R&D-based endogenous growth models, accumulation of knowledge through innovation, plays the same role as the human capital development in the Lucas setting. Regardless of their different microfoundations, the two groups of human capital and R&D-based models, share the same reduced form in the context of optimal growth theory, so long as they are subject to the Lucas assumption by which one sector specializes in only one type of capital. This implies that our results are applicable to the R&D-based models too. This includes the expanding variety model of Romer and the creative destruction model of Aghion and Howitt.

Barro and Sala-i-Martin(1995, sec. 5.2.2) show that the imbalance between physical and human capital is one of the main forces that governs the dynamics of the Lucas model. Moreover consumption and physical capital respond in an

asymmetric fashion to the extent of this imbalance. Based on this result, we apply our findings to rationalize the growth failure of oil economies. According to our empirical observations, we find an association between oil abundance and the extent of imbalance between physical and human capital in favour of the former. Using this, our results on the U-shaped path of consumption and physical capital, justify the high but unsustained levels of consumption and physical capital in oil economies. In addition, we offer a partial explanation for the poor growth performance of these economies and the adverse growth effect of oil booms.

Caballé and Santos(1993) show that in a Lucas economy, in reaction to a sudden increase in physical capital, consumption may grow at a higher rate than the investment itself. We relate this findings to account for the voracity effect, introduced in section 2.6.2, in a representative-agent framework.

Another application of the U-shaped path of consumption and physical capital, is to contrast two notions of optimality and sustainability in Lucas-type growth models in a concrete way. Optimality, according to this argument, may depart from sustainability along the transition path though growth is sustained at the steady state.

The plan of the Chapter is as follows. We present the benchmark model of a Lucas economy in section 3.2. This is followed by exploring the dynamics of consumption and physical capital in section 3.3 and local dynamics of the output growth in section 3.4. Sections 3.5 and 3.6 examine the validity of our findings when the model is extended to the cases where consumption smoothing is not strong and there is an externality from human capital accumulation respectively. We then present some of the applications of our findings in sections 3.7 and 3.8. The concluding remarks are presented at the final section.

## 3.2 The model

### 3.2.1 Specification

We present in this section a two-sector endogenous growth model in the Lucas tradition. In the Lucas model, labour in efficiency units, is reproduced in an unbounded fashion through education. Preferences are isoelastic, technology of output product is Cobb-Douglas and neither physical nor human capital are subject to depreciation. Population grows at an exogenous rate. There exists an external effect from the social stock of human capital to the productivity of individual agents and, according to a crucial assumption, education depends only on the human capital and the allocation of time between working and schooling. In other words, human capital is the only asset to be allocated across sectors. Lucas only considers the balanced growth path, along which all level variables grow at a constant rate.

As a benchmark model to begin with, we abstract from externalities from human capital accumulation. We also assume that population does not grow. Because the decentralized version of the model, in the absence of externality and other distortions, may be replicated through the planner's optimal solution, we shall limit without loss of generality our analysis to the planner's problem. In this regard our model is close to those of Barro and Sala-i-Martin(1995, sec. 5.2.2) and Arnold(2000).

Consider an economy with constant population normalized to one. The objective of the planner is to maximize intertemporal isoelastic utility of the representative agent

$$W = \int_0^{\infty} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad (3.1)$$

where  $C(t)$  denotes consumption,  $\rho > 0$  is the discount rate, and  $1/\sigma > 0$  is the intertemporal elasticity of consumption.

Output is determined by the Cobb-Douglass technology

$$Y = AK^\alpha(uH)^{1-\alpha}, \quad (3.2)$$

where  $K$  is physical capital,  $H$  denotes human capital,  $u \in [0, 1]$  is the amount of effort devoted to production of the final output, and  $\alpha \in (0, 1)$  is the elasticity of output with respect to the physical capital. The final good is consumed, invested to accumulate physical capital or to replace depreciated old capital. Hence

$$\dot{K} = Y - C - \delta K. \quad (3.3)$$

Although in some literature e.g. Bond et al.(1996, p.153) and Turnovsky(2000, p.230), disinvestment is feasible, in line with mainstream, we suppose that investment is irreversible and hence gross investment is nonnegative<sup>1</sup>. This requires  $\dot{K} + \delta K \geq 0$ .

The representative producer-consumer supplies one unit of labour inelastically and lives forever. S/he devotes  $1 - u$  fraction of her/his effort to the development of human capital, referred to as education. Then

$$\dot{H} = B(1 - u)H - \delta H, \quad (3.4)$$

where  $B > 0$  is the productivity of education. This indicates that development of human capital only depends on the proportion of time spent on education and the pre-accumulated human capital. In particular it does not require the units of final output and hence physical capital. We also assume that both types of capital depreciate at a common rate<sup>2</sup>.

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<sup>1</sup>See Barro and Sala-i-Martin(1995, ch.5) for an extensive discussion on the issue.

<sup>2</sup>The depreciation of human capital on the individual level reflects depreciation caused by forgetting. It also reflects the imperfections in the intergenerational transmission of skills.

### 3.2.2 Optimal growth and the steady state

The central planner chooses level of  $C$  and  $u$  such that  $W$  is maximized subject to the law of motion of  $K$  and  $H$  and given values for  $K(0)$  and  $H(0)$ . Two decisions are made here: the allocation of output between consumption and investment in physical capital, and the allocation of time between working and education.

An optimal solution for this economy is therefore a path  $\{C(t), K(t), H(t), u(t)\}$  which maximizes (3.1) subject to the constraints (3.3) and (3.4) and transversality conditions corresponding to  $K$  and  $H$ . We firstly concentrate on the steady state where the growth rates are balanced.

**Definition 5** *The steady state is a path, along which,  $C$ ,  $Y$ ,  $K$  and  $H$  grow at constant rates and  $u$  remains unchanged.*

Lucas shows that in the absence of population growth and externality, all level variables share a common steady state rate of growth equal to

$$\tilde{g} = B(1 - \tilde{u}) - \delta, \quad (3.5)$$

where a tilde over a variable refers to its steady state.

It is more convenient therefore to transform the variables into a set of new variables  $z \equiv Y/K$  and  $\chi \equiv C/K$  that are stationary at the steady state. These, with  $u$ , form *fundamentals* of the model. A steady state, or balanced growth path, can then be defined as a situation where  $\dot{z} = \dot{\chi} = \dot{u} = 0$ , so that consumption and the two types of capital grow at a common rate while the work effort will be constant.

The problem of optimal growth of the Lucas model, has been solved by Barro and Sala-i-Martin(1995, sec.5.2), Ortigueira(1998) and Arnold(2000). The dynamics of the economy can be expressed by a set of linear growth differential

equations (see appendix B.2 for details) where for any variable like  $y$ ,  $g_y \equiv \dot{y}/y$  denotes its exponential rate of growth and  $\lambda = (\alpha - 1)B/\alpha$ ,

$$\begin{bmatrix} g_z \\ g_\chi \\ g_u \end{bmatrix} = \begin{bmatrix} \alpha - 1 & 0 & 0 \\ \alpha/\sigma - 1 & 1 & 0 \\ 0 & -1 & B \end{bmatrix} \begin{bmatrix} z \\ \chi \\ u \end{bmatrix} - \begin{bmatrix} \lambda \\ (\rho + \delta)/\sigma - \delta \\ \lambda \end{bmatrix}. \quad (3.6)$$

In a compact form we have

$$[g_z, g_\chi, g_u]^T = Mx - [\lambda, (\rho + \delta)/\sigma - \delta, \lambda]^T,$$

where  $x = [z, \chi, u]^T$  is the vector of fundamentals. Since the matrix of coefficient,  $M$ , has a nonzero determinant, equal to  $(\alpha - 1)B$ , it is nonsingular. As a result there exists a unique steady state defined by<sup>3</sup>

$$\tilde{x} = M^{-1} [\lambda, (\rho + \delta)/\sigma - \delta, \lambda]^T.$$

Regarding the properties of the system (3.6), one should note that the model has a block recursive structure. The dynamics of the output-capital ratio,  $z$  is independent of  $\chi$  and  $u$ . Further, the dynamics of the consumption-capital ratio,  $\chi$  is independent of  $u$ . This substantially simplifies the analysis of the transitional dynamics.

Solving for the steady state value of the fundamental variables, one obtains

$$\begin{aligned} \tilde{z} &= B/\alpha, \\ \tilde{\chi} &= B/\alpha - (B - \rho - \delta)/\sigma - \delta, \\ \tilde{u} &= 1 - (B - \rho - \delta)/B\sigma - \delta/B. \end{aligned} \quad (3.7)$$

From this, in conjunction with (3.6), one can reproduce Eqs. (5.31) - (5.33)

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<sup>3</sup>See Bond et al.(1996) on the existence and uniqueness in a general setting.



of Barro and Sala-i-Martin(1995) as

$$(g_z, g_\chi, g_u)^T = M \cdot (z - \tilde{z}, \chi - \tilde{\chi}, u - \tilde{u})^T. \quad (3.8)$$

Plugging the steady state value of work effort,  $\tilde{u}$  into (3.5) gives the balanced rate of growth of the economy as

$$\tilde{g} = (B - \rho - \delta)/\sigma. \quad (3.9)$$

Sustainability of the long run growth requires  $\rho + \delta < B$  and the transversality condition corresponding to  $K$  implies  $\tilde{g} < B - \delta$ . Two requirements are combined in the following condition.

**Condition 5** *For the balanced rate of growth to sustain and also to meet the transversality conditions, one requires*

$$0 < B - \rho - \delta < \sigma(B - \delta). \quad (3.10)$$

This is also the sufficient condition for  $\tilde{\chi} > 0$ , and necessary and sufficient condition for  $\tilde{u} \in (0, 1)$ , i.e. for the steady state to be well defined.

The effect of changes in parameters of the model on the steady state values  $(\tilde{z}, \tilde{\chi}, \tilde{u})$  and  $\tilde{g}$  is reported in Mulligan and Sala-i-Martin(1993, table 1).

### 3.2.3 Local dynamics around the steady state

Local dynamics of the system (3.6) in the vicinity of the steady state (3.7), can be found by linearizing the system of differential equations around  $\tilde{x}$ . This can be summarized as

$$\dot{x} \simeq \tilde{M} \cdot (x - \tilde{x}), \quad (3.11)$$

where  $\widetilde{M} = [\widetilde{m}_{ij}]$  is a  $3 \times 3$  Jacobian matrix for which we have  $\widetilde{m}_{ij} = m_{ij} \cdot \widetilde{x}_i$  and  $m_{ij}$  is the entry on  $i$ -th row and  $j$ -th column of  $M$ .

By definition  $\widetilde{M}$  is lower triangular like  $M$  and the eigenvalues, defined by  $\lambda(\cdot)$ , coincide with its diagonal entries as

$$\lambda(\widetilde{M}) = \{ (\alpha - 1)B/\alpha, B/\alpha - (B - \rho)/\sigma, B - (B - \rho)/\sigma \}.$$

Given (3.10) and the range of parameters of the model, there is only one negative eigenvalue,  $\lambda = (\alpha - 1)B/\alpha$ , whose magnitude determines the speed of convergence in the vicinity of the steady state. This shows that (3.11) describes a one dimensional stable saddle path.

Starting from a given initial value of the average productivity of physical capital  $z(0) = z_0$ , the stable dynamic adjustment path is described around the steady state by

$$\begin{aligned} z_t - \widetilde{z} &= (z_0 - \widetilde{z}) \cdot (z_t/z_0) \exp(\lambda t), \\ \chi_t - \widetilde{\chi} &= v_2 \cdot (z_t - \widetilde{z}), \\ u_t - \widetilde{u} &= v_3 \cdot (z_t - \widetilde{z}), \end{aligned}$$

or in a compact form

$$x_t - \widetilde{x} = (z_0 - \widetilde{z})(z_t/z_0) \cdot V \exp(\lambda t),$$

where  $V = (1, v_2, v_3)^T$  is the eigenvector corresponding to the stable eigenvalue  $\lambda$ , that rules out the unstable paths.

The first equation in (3.6) presents a self contained differential equation for  $z$  which gives

$$z_t = \widetilde{z} \cdot z_0 [z_0 - (z_0 - \widetilde{z}) \exp(\lambda t)]^{-1}.$$

This, for  $z_0 < \tilde{z}$ , is logistic while for  $z_0 > \tilde{z}$ , it declines exponentially toward  $\tilde{z}$ . Moreover, since there is no restriction on the convergence of  $z$  to  $\tilde{z}$  and it attracts any  $z > 0$ , then stability is global rather than local; Arnold(2000).

We summarize our discussions about the optimal model described by Eqs. (3.1)-(3.4) in the following statements:

- There exists a unique steady state that is completely described by known parameters.
- The steady state is globally saddle stable.
- The speed of convergence,  $-\lambda = (1 - \alpha) B/\alpha$ , depends solely on technical parameters.

### 3.2.4 Policy functions and growth regimes

The one-dimensional saddle stable arm can be denoted by the curve

$$(z, \chi(z), u(z)) : R_+ \longrightarrow R_+^2 \times [0, 1],$$

that passes through the steady state, i.e.  $\chi(\tilde{z}) = \tilde{\chi}$ , and  $u(\tilde{z}) = \tilde{u}$ . The slope of the curve at the steady state also distinguishes the stable path from unstable ones, i.e.  $\chi'(\tilde{z}) = v_2$ , and  $u'(\tilde{z}) = v_3$ , where

$$v_2 = \left( \frac{1 - \alpha/\sigma}{\tilde{\chi} - \lambda} \right) \tilde{\chi}, \quad \text{and} \quad v_3 = \left( \frac{1 - \alpha/\sigma}{\tilde{\chi} - \lambda} \right) \tilde{u}. \quad (3.12)$$

Since according to Eq. (3.6), the dynamics of consumption and physical capital only depends on output-capital and consumption-capital ratios, from now on we only focus on the projection of the saddle path on the  $(z, \chi)$  plane.

Regarding the relative size of  $\alpha$  and  $\sigma$ , three cases are distinguished in the literature; Caballé and Santos(1993). For the immediate purpose we consider the

case where  $\alpha < \sigma$  which is more interesting in practice. The cases where  $\sigma \leq \alpha$  are discussed in section 3.5.

If  $\alpha < \sigma$ , the locus  $\dot{\chi} = 0$  is upward sloping with slope  $1 - \alpha/\sigma > 0$ . For  $0 \leq z_0 < \tilde{z}$  we then have

$$(\rho + \delta)/\sigma - \delta + (1 - \alpha/\sigma)z < \chi(z) < \tilde{\chi}.$$

Since the locus  $\dot{z} = 0$  is vertical and stable, for  $z_0 < \tilde{z}$  (respectively  $z_0 > \tilde{z}$ ),  $z(t)$  converges monotonically to  $\tilde{z}$  from left (respectively from right). The phase diagram in this case is depicted in the upper panel of figure 3.1.<sup>4</sup>

Mulligan and Sala-i-Martin(1993) identify three forces that govern the dynamics of the model off the steady state. Without loss of generality consider the case where  $z_0 < \tilde{z}$ . This means that the economy is endowed with a relatively scarce level of human capital and the productivity of physical capital is low.

Firstly, owing to the global stability of the steady state, there exists a convergence mechanism that forces the economy from imbalances between physical and human capital to the balanced path. This is described by the dynamics of  $z$  alone.

In addition, there are two other forces. Convergence may be supported through consuming more and investing less in physical capital or via faster accumulation of human capital by devoting more time to schooling and working less. The stronger the consumption-smoothing effect, people dislike the former option. On the other hand, when  $H/K$  is low, the wage rate in the output sector is high, discouraging agents from schooling by lowering  $u$ .

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<sup>4</sup>In comparison with figure 5.4 of Barro and Sala-i-Martin(1995), this is augmented with the required details to enable us extracting the transition path of rate of growth of consumption and physical capital along the whole possible range of output-capital ratio.

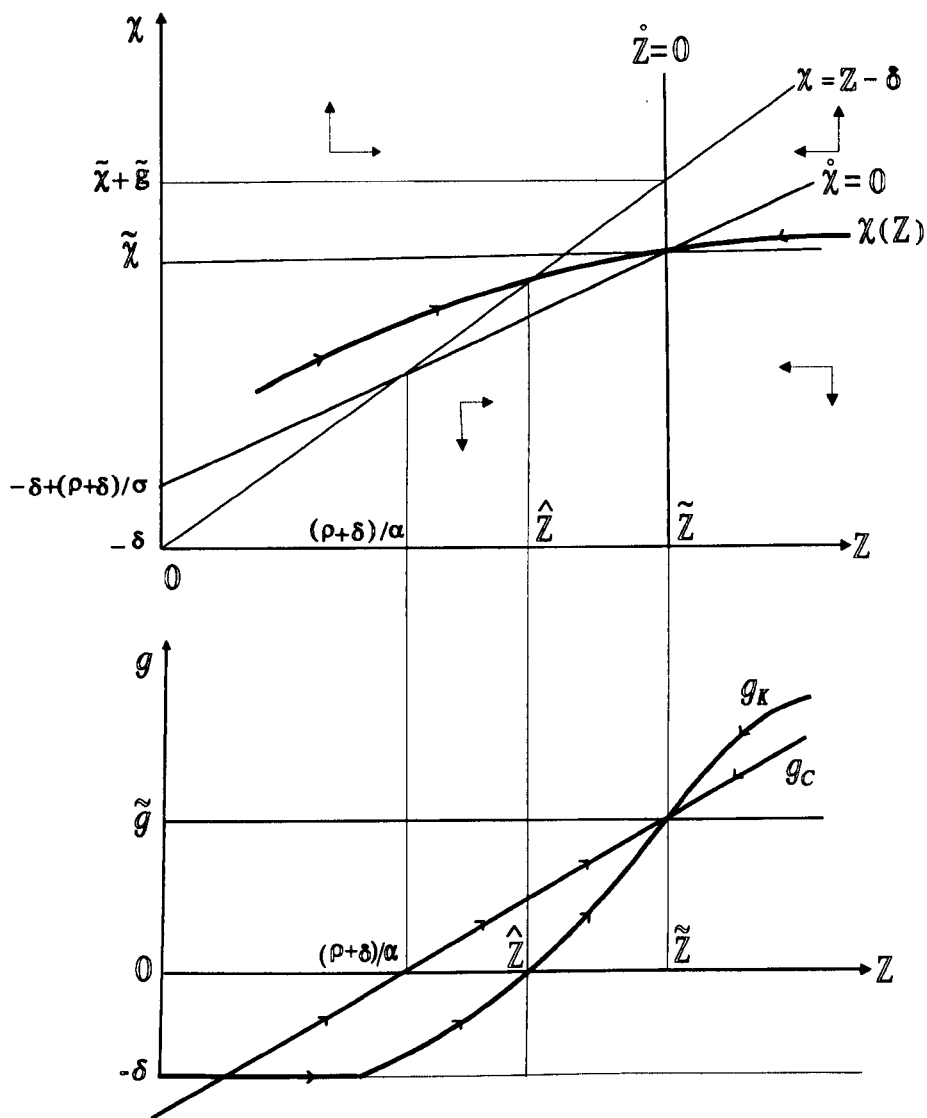


Figure 3.1: Phase diagram of  $z$  and  $\chi$  (upper panel) and  $g_C$  and  $g_K$  (lower panel)

When the technology of producing the final output is Cobb-Douglas, like the model at hand, the relative size of the intertemporal elasticity of substitution in consumption and the share of physical capital, account for the growth regime. The smaller  $\sigma$ , the stronger is the second effect. On the other hand, the larger  $\alpha$ , the more powerful is the third force. The relative position of these two parameters determines how these three forces interact and which effect dominates.

If  $\alpha < \sigma$ , the consumption-smoothing effect dominates. Low  $z$  corresponds with low  $\chi$  and  $u$ , meaning that the policy functions are upward sloping. In this case, the transition from relatively high levels of physical capital is accomplished through more schooling rather than higher saving.

When  $\alpha = \sigma$ , the two forces cancel out and only the convergence effect takes place. In this case the policy function is horizontal meaning that a constant proportion of time is devoted to education and the human capital grows at an exogenous rate. The Lucas model, as a result, replicates the dynamics of the Ramsey model where education takes the role of exogenous technical progress.

Finally if  $\alpha > \sigma$ , low values of  $z$  correspond with high  $\chi$  and  $u$ , and policy functions are downward sloping. In this case, besides the convergence effect, transition to the steady state is governed mostly by changes in consumption patterns rather than by working effort. In particular, the dynamics of an economy starting from  $z_0 < \tilde{z}$  to the steady state in this regime is accomplished through less saving rather than supplying more effort.

These three regimes are called the *normal*, *exogenous* and *paradoxical growth* cases respectively; Caballé and Santos(1993) and Ladron-de-Guerara et. al. (1997). Although, according to the empirical evidence, the Cobb-Douglas technology is in favour of the normal case, in a more general setting the numerical

exercise of Caballé and Santos do not rule out other possibilities. Condition (3.10) on the other hand, imposes a lower bound on  $\sigma$  equal to  $1 - \rho/(B - \delta)$ . This further limits the possibility of other cases in the Cobb-Douglas framework. For the rest of this study we follow the normal case until section 3.5 where other growth regimes are explored in more detail.

### 3.3 U-shaped paths of consumption and physical capital

In this section we fully explore the transition dynamics of consumption and physical capital.

#### 3.3.1 Dynamics of consumption

Rate of growth of consumption

$$g_C(z) = (\alpha z - \rho - \delta)/\sigma,$$

mimics the pattern of  $z$  along the transition. Since  $z$  is defined on the whole nonnegative real axis, it is possible for  $z_0$  to fall short of  $(\rho + \delta)/\alpha$ . This implies  $g_C < 0$  and consumption thus falls along the transition. Since  $z$  adjusts monotonically to  $\tilde{z}$ ,  $z(t)$  eventually passes its threshold and consumption begins to rise. In this case consumption exhibits a nonmonotonic time profile. It firstly declines and reaches its minimum at

$$t_1 \equiv z^{-1}((\rho + \delta)/\alpha),$$

and then rises with an increasing rate.

The time path of consumption is different when  $(\rho + \delta)/\alpha \leq z_0 < \tilde{z}$ . In this case, it grows at an increasing rate. Finally when  $z_0 > \tilde{z}$ , consumption grows

along the transition at a decreasing rate. In all cases because of sustainability of growth, i.e. condition (3.10),  $g_C$  eventually exceeds zero and approaches its long-run value on the balanced growth path,  $\tilde{g} > 0$ . The findings are summarized as follows:

**Proposition 13** *In the Lucas model, described by Eqs. (3.1)-(3.4), for  $\alpha < \sigma$ , the transition of consumption is governed through the following pattern:*

(i) *If  $z_0 < (\rho + \delta)/\alpha$ ,  $C(t)$  falls on  $0 \leq t \leq t_1$ . It rises with an increasing rate afterwards. This results in a U-shaped path for consumption whose minimum equals  $C(t_1)$ . The stronger the consumption smoothing effect, i.e. larger  $\sigma$ , the shallower is the U-path.*

(ii) *If  $(\rho + \delta)/\alpha < z_0 < \tilde{z}$ ,  $C$  grows with an increasing rate along the off-balanced path.*

(iii) *If  $z_0 > \tilde{z}$ ,  $C$  rises with a decreasing rate along its transition path.*

### 3.3.2 Dynamics of physical capital

The locus  $\dot{\chi} = 0$ , with a positive intercept, is flatter than the 45° line and crosses it at  $z = (\rho + \delta)/\alpha$  which by (3.10) is less than  $\tilde{z}$ . On the other hand, by (3.7),  $\tilde{\chi} < \tilde{z}$ . This gives  $(\rho + \delta)/\alpha < \tilde{\chi} < \tilde{z}$ .

For  $0 \leq z < \tilde{z}$ ,  $\chi(z)$  lies between  $\dot{\chi} = 0$  and  $\chi = \tilde{\chi}$ . Moreover according to the direction of movements in the phase diagram (see figure 1),  $\chi(z)$  is strictly increasing. This implies

$$(\rho + \delta)/\alpha < \hat{z} < \tilde{\chi} + \delta, \quad (3.13)$$

where  $\hat{z} = \arg\{\chi(z) = z - \delta\}$ . In contrary to the Ramsey model, this indicates that there are two different thresholds corresponding to consumption and physical capital in the Lucas framework.



Let  $d(z)$  measures the distant between  $\chi(z)$  and the  $\dot{\chi} = 0$  locus, i.e.  $d(z) \equiv \chi(z) - (1 - \alpha/\sigma)z + \delta - (\rho + \delta)/\sigma$ , we have  $d(z) > 0$  for  $z < \tilde{z}$  and  $d(\tilde{z}) = 0$  (see the upper panel of figure 1). Simple manipulation gives  $g_K(z) = g_C(z) - d(z)$ . This implies  $g_K < g_C$  for  $z < \tilde{z}$ .

### 3.3.3 Phases of transition

In general, when the average productivity of physical capital  $z$  is far short of its steady state  $\tilde{z}$ , four episodes occur. First, when  $0 < z(t) < (\rho + \delta)/\alpha$  both consumption and physical capital fall. During the second episode, when  $(\rho + \delta)/\alpha < z(t) < \hat{z}$ , consumption rises but physical capital still decumulates, i.e.  $g_K(z) < 0 < g_C(z)$ . In the third phase, when  $\hat{z} < z(t) < \tilde{z}$ , both variables rise with different rates, i.e.  $0 < g_K(z) < g_C(z)$ . Finally, at the steady state they both grow at a common and constant rate,  $g_K(\tilde{z}) = g_C(\tilde{z}) = \tilde{g}$ .<sup>5</sup> The evolution of rates of growth of consumption and physical capital as functions of  $z$  along the stable arm is depicted in the lower panel of figure 3.1.

The transitional path of consumption and physical capital in the four episodes mentioned above are depicted in the panels of figure 3.2. The first and third panels parallel those situations of the Ramsey model where an economy approaches the steady state from above and below respectively. The lower panel also addresses endogenous growth in the Lucas model which has no counterpart in the

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<sup>5</sup>One may wonder to what extent, the possibility of lending and borrowing alters our results on the dynamics of consumption and physical capital. In the context of models similar to the one outlined here, when there is no imperfection, caused e.g. by adjustment cost or government intervention, a paper asset bears the same rate of return as the physical capital. Agents are, thus, indifferent in holding paper or physical asset. The intertemporal transfer of resources is therefore governed through the subjective discount rate,  $\rho$ , and consumption smoothing is not affected by lending and borrowing; Blanchard and Fischer(1989, sec. 2.2).

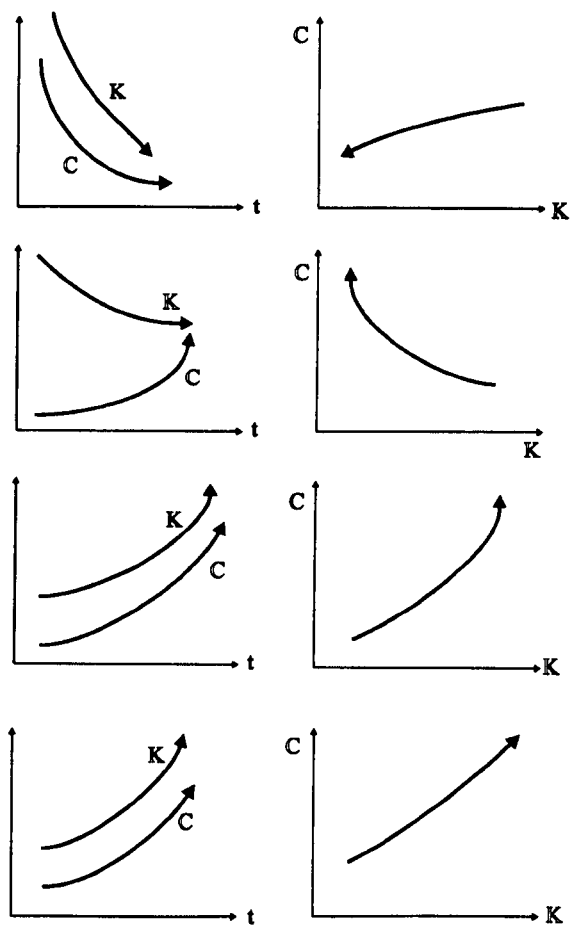


Figure 3.2: Phases of transition of consumption and physical capital

Ramsey model. The second panel, however, highlights one of the distinct features of transition of  $C$  and  $K$  in the Lucas model. In this stage, consumption rises while physical capital falls. We elaborate shortly the differences in the dynamics of consumption and physical capital in the Lucas and Ramsey model in detail.

**Proposition 14** *In the Lucas model, described by Eqs. (3.1) - (3.4), where  $\alpha < \sigma$ , the followings hold.*

*i) The rate of growth of consumption and physical capital adjust monotonously toward their common steady state  $\tilde{g}$ . They change along their transition paths in the same direction and increase with the average productivity of physical capital. During transition,  $g_K(z) \leq g_C(z)$  for  $z \leq \tilde{z}$  though as  $z$  approaches  $\tilde{z}$ , they become closer to each other.*

*ii) For  $z_0 < (\rho + \delta)/\alpha$ , consumption and physical capital display U-shaped patterns on their transition paths. They fall for a finite period and then rise. In spite of the Ramsey model, here there are two different thresholds determining whether consumption and physical capital fall or rise during the adjustment process. The minimum of consumption always occurs before that of physical capital, i.e.  $\arg \min\{C(t)\} < \arg \min\{K(t)\}$ .*

*iii) According to the sequence of stages of transition, there exists a situation where a boost in consumption coincides with decumulation of physical capital.*

The occurrence of a negative rate of growth on the transition depends on the size of the initial value of  $z = (uH/K)^{1-\alpha}$  which is a measure of imbalance between physical and human capital relative to its steady state. The more scarce the level of skills with respect to the equipment and plant, the lower  $z$ , and it is more likely for  $z$  to fall behind the thresholds corresponding to  $C$  and  $K$  resulting in their U-shaped path during transition. Hence the falling of consumption

and physical capital depends heavily on the extent of imbalance between the two sectors. Furthermore the less productive the technology of human capital accumulation, the longer is the falling period.

The likelihood of the occurrence of the falling period for an economy which is subject a the high ratio of physical to human capital depends on the magnitude of the effective discount rate  $\rho + \delta$  relative to the productivity of education technology,  $B$  on one hand and curvature of the policy function  $\chi(z)$  on the other. The higher  $(\rho + \delta)/B$ , consumption is more likely to go down during its transition, making the first stage longer. The flatter the policy function,  $\chi(z)$  on the other hand, the farther is  $\hat{z}$  from its lower bound,  $(\rho + \delta)/B$  and closer it is to its upper bound,  $\tilde{\chi}$  implying that the second period lasts longer. In addition, the closer  $\hat{z}$  to  $\tilde{z}$ , the longer the third period lasts.

The range of benchmark values of parameters in the literature suggests that the above mentioned threshold might be close to  $\tilde{z}$ . For example the corresponding figure in Mulligan and Sala-i-Martin(1993) and Barro and Sala-i-Martin(1995, sec. 5.2.2) is respectively 75 and 63.5 percent. This states that consumption and physical capital might fall even when the economy is quite close to the steady state.

### 3.3.4 Numerical simulation

We consult the time elimination method, introduced by Mulligan and Sala-i-Martin (1993), to solve the system of growth differential equations described by Eq. (3.6). For this purpose we apply the set of their base line parameters:  $A = 1$ ,  $B = 0.12$ ,  $\alpha = 0.5$ ,  $\delta = 0.05$ ,  $\rho = 0.04$  and  $\sigma = 2$ . These give the steady state as  $\tilde{z} = 0.24$ ,  $\tilde{\chi} = 0.175$ ,  $\tilde{u} \cong 0.458$ , and  $\tilde{g} = 1.5\%$ .

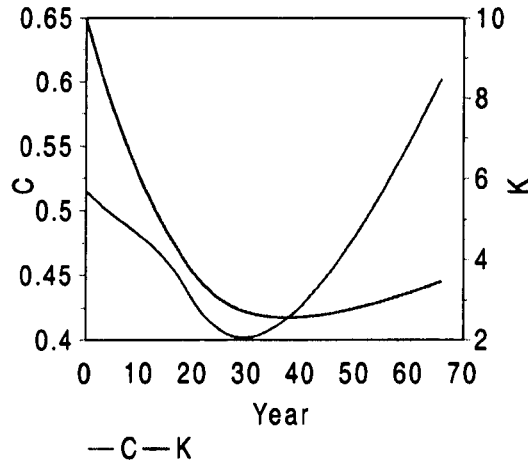


Figure 3.3: Time profile of consumption and physical capital

We solve the system backwards by taking the steady state as the initial condition while by choosing eigenvector corresponding to the stable eigenvalue  $\lambda = -0.12$ , the solution traces the stable arm in  $(z, \chi, u)$  space. The algorithm is described in appendix B.1. The results are depicted in figures 3.3 and 3.4.

Figure 3.3 refers to the time profiles of consumption and physical capital when the average productivity of capital is short of its steady state. The U-shaped path of consumption and physical capital is apparent from this figure. As one can observe, the falling period of  $C$  and  $K$  last more than 29 and 37 years respectively.

The first, second and third episodes occur in periods  $[0, 30)$ ,  $[30, 38)$ ,  $[38, 67)$  respectively. These are depicted in the  $(C, K)$  space in figure 3.4 where the curve bends during the second stage of transition. Our numerical simulation shows that the findings are valid for a wide range of parameters as long as they are in

their meaningful ranges and  $\alpha < \sigma$ .

### 3.3.5 Overaccumulation of capital in the Ramsey model

Here we briefly compare briefly here our findings with the Ramsey model where capital exceeds the modified golden rule and both consumption and capital decline monotonically toward their steady state.

The intuition behind falling are similar in the Lucas and Ramsey models, though they produce different transition paths. In both settings, according to the Euler equation, consumption falls when marginal productivity of (physical) capital is short of the effective discount rate. Falling of (physical) capital also, in both cases, is attributed to the case where consumption exceeds net output.

In the Ramsey model, falling of  $C$  and  $K$  coincide so that if capital exceeds the modified golden rule both consumption and capital decline monotonically toward their steady state. In the Lucas model instead, there are two different thresholds associated with falling of consumption and physical capital. Moreover steady state occurs where the *rate of growth* of the level variables, rather than their *level* themselves, are steady.

The cause of decline of  $C$  and  $K$  in the Lucas model is attributed to the high ratio of  $K$  to  $H$  which happens either due to the relative abundance of physical capital or the shortage of human capital. This measure of *sectorial imbalance* is more comprehensive than the mere overaccumulation of capital in the Ramsey model. Moreover, here the productivity of human capital accumulation is the key factor that determines, due to the intersectoral imbalances, how long the economy will stay in the falling period.

Finally, the time profile of consumption and physical capital are nonmono-

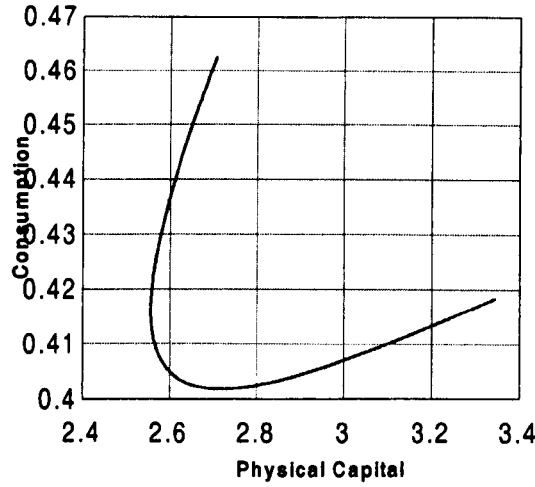


Figure 3.4: Evolution of consumption and physical capital in the  $(K, C)$  plane

tonic here. They firstly fall and then gradually rise at an increasing rate. This happens in the presence of sustainability of growth in the steady state while there is no endogenous growth in the Ramsey model.

To make the comparison clearer, let us consider a Ramsey economy where labour grows at an exogenous rate equal to  $\tilde{g}$  in (3.9). At the steady state the Ramsey model reproduces a Lucas economy. Along the transition, consumption and physical capital comove in the same direction monotonically toward their steady state path. They both rise when the rate of return exceeds the effective discount rate,  $\rho + \delta$  and fall when the reverse is the case. This replicates stages 1, 3 and 4 above but can not generate stage 2 in which the rise of consumption coincides with the decline of physical capital. This relates to the scope of section 3.5.

### 3.4 Local dynamics of output growth

Rate of growth of output along the transition path is equal to:

$$g_Y(z) = -(\lambda + \delta) + \alpha z - \chi(z). \quad (3.14)$$

This means,  $g_Y$  has no component along the  $u$  direction and the *isogrowth* lines of  $Y$  in  $(z, \chi)$  plane

$$\chi(z) = -(\lambda + \delta) - \overline{g_Y} + \alpha z,$$

are those with slope  $\alpha$  where  $\overline{g_Y}$  is the constant rate of output growth across the line. Moreover, the higher their intercepts the lower is the rate of growth of output along the isogrowth lines.

The position of isogrowth lines relative to the policy function  $\chi(z)$ , determines the transitional dynamics of output growth. In the vicinity of the steady state,  $v_2 = (1 - \alpha/\sigma) \tilde{\chi}/(\tilde{\chi} - \lambda)$  gives the slope of the policy function  $\chi(z)$  in the  $(z, \chi)$  plane. Along the whole transition path however the slope is given by

$$\chi'(z) = \frac{\dot{\chi}}{\dot{z}} = \frac{(\alpha/\sigma - 1)z + \chi - \rho/\sigma + (1 - 1/\sigma)\delta}{(\alpha - 1)z - \lambda} \cdot \frac{\chi}{z},$$

which by mere analytical methods, one cannot decide whether is steeper or flatter than the isogrowth lines. We limit our analysis therefore only to the *local* dynamics of output growth.

Concerning the slope of isogrowth lines relative to  $\chi(z)$  in the neighbourhood of  $\tilde{z}$ , four cases can be distinguished:

**Case 6**  $\alpha \geq 1 - \alpha/\sigma$ .

The isogrowth lines are steeper than the  $\dot{\chi} = 0$  locus and hence  $\chi(z)$  itself in this case. They, thus, cross the policy function only once. The lower values



of  $\chi$ , correspond to the isogrowth lines that lie farther to the left. Since  $\chi(z)$  is increasing, this introduces a one-to-one relationship between  $z$  and  $g_Y$ , implying that  $g_Y(z)$  is increasing. The position of isogrowth lines in the phase diagram in this case and also  $g_Y(z)$  is depicted in panels (i) and (ii) of figure 3.5 respectively.

**Case 7**  $v_2 < \alpha < 1 - \alpha/\sigma$ .

The isogrowth lines are steeper than  $\chi$  in the vicinity of the steady state, but flatter than the  $\dot{\chi} = 0$  locus here. A smooth enough policy function, is crossed only twice by an isogrowth line with low enough intercept. In particular, given the set of parameters, there is a unique  $\bar{z} < \tilde{z}$  such that  $\alpha = \chi'(\bar{z})$  and  $g_Y(\bar{z}) = \min\{g_Y(z) : z > 0\}$ . This case is depicted in panels (iii) and (iv).

**Case 8**  $v_2 = \alpha$ .

The isogrowth line passing through the  $(\tilde{z}, \tilde{\chi})$  point, in this case, is tangential to the policy function. This implies that the rate of growth of output is minimized at the steady state, i.e.  $g_Y(\tilde{z}) = \min\{g_Y(z) : z > 0\}$ ; panels (v) and (vi).

**Case 9**  $v_2 > \alpha$ .

The isogrowth lines are flatter than  $\chi(z)$  in a neighbourhood of the steady state. Since  $\chi$  is concave in  $z$  in the vicinity of  $\tilde{z}$ , the isogrowth lines with sufficiently low intercepts cross it twice. In particular there exists  $\bar{z} > \tilde{z}$  such that  $\alpha = \chi'(\bar{z})$  and  $g_Y(\bar{z}) = \min\{g_Y(z) : z > 0\}$ . This case is depicted in panels (vii) and (viii).

Given the parameters of the model (except  $\sigma$ ), we can characterize the above cases according to the size of the intertemporal elasticity of consumption. Considering that for  $\chi(z)$  to be upward sloping, we have assumed  $\alpha < \sigma$ , then case

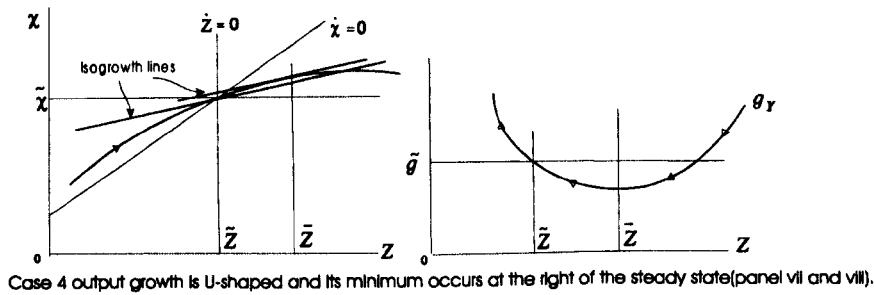
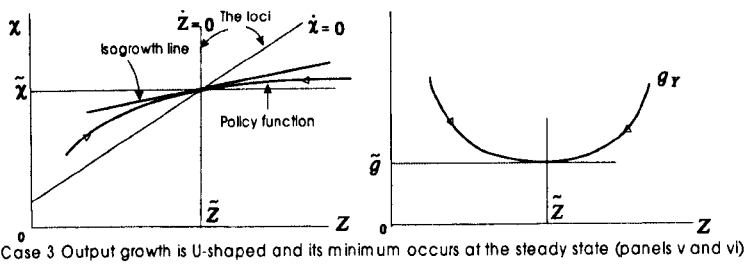
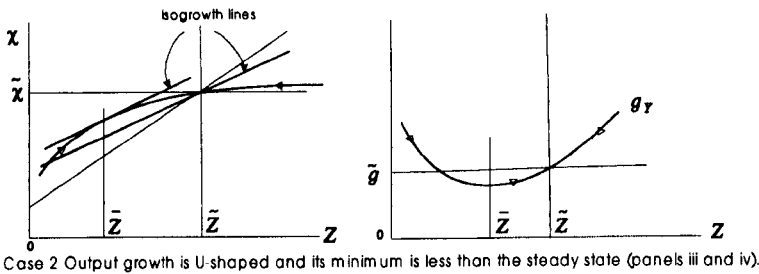
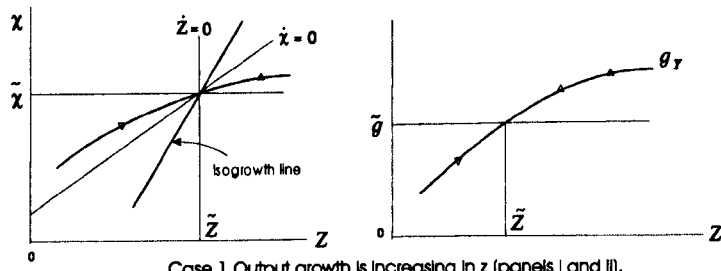


Figure 3.5: Classification of the dynamics of output growth

7 corresponds to  $\alpha < \sigma \leq \alpha/(1 - \alpha)$ . The condition for case 8, i.e.

$$1/\alpha - 1/\sigma = 1 - \lambda/\tilde{\chi},$$

defines a quadratic polynomial equation in terms of  $\sigma$  in the form  $a\sigma^2 - b\sigma + (B - \rho - \delta) = 0$  where

$$\begin{aligned} a &= \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{B}{\alpha} - \delta - B \right), \\ b &= \left( \frac{1 - \alpha}{\alpha} \right) (B - \rho - \delta) - B/\alpha + \delta. \end{aligned}$$

This has only one root - called  $\tilde{\sigma}$  - greater than  $\alpha/(1 - \alpha)$ .

Hence for  $\sigma > \alpha/(1 - \alpha)$ ,  $g_Y$  is U-shaped in  $z$ . Its minimum occurs at the left or right of  $\tilde{z}$ , depending on whether  $\sigma < \tilde{\sigma}$  or  $\sigma > \tilde{\sigma}$  respectively. We summarize our findings in the following statement.

**Proposition 15** *Let  $\alpha < \sigma$ . Local dynamics of the rate of growth of output in the vicinity of the steady state depends on the size of the intertemporal elasticity of consumption. When  $\alpha < \sigma \leq \alpha/(1 - \alpha)$ ,  $g_Y$  is increasing in  $z$ , while for  $\sigma > \alpha/(1 - \alpha)$ , it is U-shaped. As  $\sigma$  increases, the minimum of  $g_Y(z)$  corresponds with the higher level of  $z$ . In particular there exists a unique  $\tilde{\sigma} > \alpha/(1 - \alpha)$  such that*

$$\arg \min g_Y(z) \leq \tilde{z} \iff \sigma \leq \tilde{\sigma}$$

To give an idea of how the size of  $\sigma$  classifies the dynamics of  $g_Y$ , we consult the baseline setting of Mulligan and Sala-i-Martin(1993) where  $\alpha = 0.5$ ,  $\delta = 0.05$ ,  $A = 1$ ,  $B = 0.12$  and  $\rho = 0.04$ . Solving the equation  $\alpha = v_2$  for  $\sigma$ , one obtains  $\tilde{\sigma} = 3$ . Hence for  $0.5 < \sigma \leq 1$ ,  $g_Y(z)$  is increasing, while for  $\sigma > 1$ , It is U-shaped. For  $\sigma = 3$ , the minimum of  $g_Y$  occurs at  $\tilde{z} = 0.24$ , while for  $\sigma \leq 3$ ,

we have  $\arg \min g_Y(z) \leq 0.24$ . In Barro and Sala-i-Martin(1995, p.191), the only difference in the baseline parameters are  $\rho = 0.02$  and  $B = 0.11$ . This gives  $\tilde{z} = 0.22$  and  $\tilde{\sigma} = 3.3$ . Hence still for  $\sigma > 1$ ,  $g_Y$  is U-shaped but we have  $\arg \min g_Y(z) \leq 0.22 \Leftrightarrow \sigma \leq 3.3$ .

The above analysis clarifies the ambiguity of the dynamics of  $g_Y$ , stated by Barro and Sala-i-Martin. It identifies the case where  $g_Y(z)$  is increasing as a new regime that has not been addressed in the literature. Moreover it delivers the U-shaped feature of  $g_Y$  in a clearer way by means of analytical methods.

### 3.5 Exogenous and Paradoxical regimes

In the exogenous growth regime, i.e. when  $\alpha = \sigma$ , convergence of  $z_0$  toward  $\tilde{z}$  is the only governing force of the dynamics of the economy. The  $\dot{\chi} = 0$  locus, in this case, is horizontal meaning that,

$$\chi(z) = \tilde{\chi} = (\rho + \delta)/\alpha - \delta, \quad \text{for } z > 0.$$

This, by the third equation in (3.8) implies  $u(z) = \tilde{u}$  for all values of  $z$  too. The transition path, thus, is a horizontal curve parallel to the  $z$  axis:

$$(z, \frac{\rho + \delta}{\alpha} - \delta, 1 - \frac{B - \rho - \delta}{B\alpha} - \frac{\delta}{B}).$$

The Lucas model, in the exogenous regime replicates dynamics of the Ramsey model where human capital development acts like an exogenous technological progress. The production function (3.2), reduces in this case to

$$Y_t = a \exp[\tilde{g}(1 - \alpha)t] \cdot K_t^\alpha, \quad a \equiv A(\tilde{u}H_0)^{1-\alpha}.$$

The main difference with the Ramsey model, in this case, is that the rate of technological progress is determined by the optimal allocation of time between

working and schooling.<sup>6</sup> This can be called an endogenous growth prototype of the Ramsey model.

Two thresholds for falling of  $C$  and  $K$ , in this case, coincide and the second stage of transition disappears. Since  $\chi$  is constant, we have  $g_K(z) = g_C(z)$  for all values of  $z$ . For  $z_0 < (\rho + \delta)/\alpha$ , consumption and physical capital still exhibit U-shaped profiles along their transition paths. In contrast with the normal regime, minimums of  $C$  and  $K$  now occur at the same time and  $C(t)$  and  $K(t)$  are parallel.

Using the same approach, one may conclude that in the paradoxical regime, i.e. where  $\sigma < \alpha$ , both  $\chi(z)$  and  $u(z)$  are decreasing. The phase diagram in the  $(z, \chi)$  plane shows that  $\chi(z)$  crosses the 45° line at a point at which  $z < (\rho + \delta)/\alpha$ . This means that the threshold for falling of physical capital,  $\hat{z}$ , in this regime is less than that of consumption,  $(\rho + \delta)/\alpha$ . As a result, in the second stage falling of consumption coincides with rising of physical capital while other stages occur in a similar fashion with the normal regime.

The following proposition summarizes these findings

**Proposition 16** *In the Lucas model, described by Eqs. (3.1)-(3.4), when the marginal productivity of physical capital is sufficiently low, four stages occur in transition of consumption and physical capital.*

(i) *In the first stage both consumption and physical capital fall. Duration of this stage, when  $\alpha \leq \sigma$  is equal  $t_1 = z^{-1}((\rho + \delta)/\alpha)$ , while for  $\sigma < \alpha$ , this stage lasts  $t_2 = z^{-1}(\arg \chi(z) = z - \delta)$ .*

(ii) *In the second stage, if  $\alpha < \sigma$ , the rise of consumption coincides with the*

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<sup>6</sup>The model also differs in two other aspects from the Ramsey model. Consumption is limited to be proportional with physical capital along the whole adjustment period. The consumption smoothing effect is also weak.

*fall of physical capital whereas for  $\sigma < \alpha$ , consumption declines when physical capital rises. This stage does not exist if  $\sigma = \alpha$ . The second stage for  $\alpha < \sigma$  terminates when  $t = t_2$ . In the case where  $\alpha > \sigma$ , it ends when  $t = t_1$ .*

*(iii) In the third stage, both consumption and physical capital rise. If  $\alpha < \sigma$ , consumption grows faster than physical capital while for  $\sigma < \alpha$ , the reverse is true. In the case where  $\sigma = \alpha$ , both variables grow at the same rate. Duration of this stage, for  $\sigma \leq \alpha$  is equal  $(t_2, +\infty)$  while for  $\sigma < \alpha$ , the time span of the third stage is equal  $(t_1, +\infty)$ .*

*(iv) Finally, in the fourth stage, both variables grow at their common long run rates.*

Phase diagram reveals that the policy function,  $\chi(z)$  is flatter than the locus  $\dot{\chi} = 0$  in both regimes. One can conclude from the analysis in section 3.5 that, each isogrowth line crosses the policy function only once. Moreover lower values of  $z$  correspond with higher isogrowth lines associated with lower rates of growth. This introduces an increasing transition pass for rate of output growth,  $g_Y(z)$  in the exogenous and paradoxical regimes which is valid for the whole values of  $z > 0$ . This, in a complementary way, extends the region obtained in proposition 15. The advantage of this result is that it unveils the dynamics of output growth, for the case where  $\sigma \leq \alpha$ , in a global rather than local fashion.

**Conclusion 8** *Let  $\sigma \leq \alpha$ . Transitional dynamics of the rate of growth of output is increasing in  $z$ . In other words  $g'_Y(z) > 0$ , for  $z > 0$ .*

### 3.6 The effect of human capital externality

Lucas suggests that the accumulation of human capital has an external effect on productivity of labour. An individual's productivity rises if others are more productive. He adds, therefore, an external effect for the economy's average level of human capital,  $\bar{H}$  to the production function (3.2):

$$Y = AK^\alpha(uH)^{1-\alpha}\bar{H}^\gamma. \quad (3.15)$$

The over bar on  $H$  indicates that agents take this quantity as given in their decision. By introducing the externality effect, the competitive equilibrium departs from being necessarily Pareto optimal. For a social planner,  $H$  has the exponent  $1 - \alpha + \gamma$  because besides its direct effect, she would take into account that human capital development raises output through externality. In the decentralized economy however, the representative agent regards  $\bar{H}$  as a parameter beyond her decision.

It is the external effect,  $\gamma > 0$  that distinguishes the social valuation of human capital accumulation from its private valuation. We consider these two cases separately in an exposition similar to that of Garcia-Castrillo and Sanso(2000).

#### 3.6.1 The decentralized economy

Consider a competitive economy populated with a mass of households; each supplies one unit of labour inelastically at a skill level  $H$ . Labour can be employed in a fraction  $u$  in the product market and obtains a wage  $w$  per unit of skill that is taken as given. S/he also devotes a fraction  $1 - u$  of time to education, improving his skill level in the form expressed in (3.4). The household is also endowed with a stock of assets  $K$  bearing the market interest rate  $r$ .

The income received in the form of labour and capital income is devoted to consumption,  $C$  and gross saving,  $dK/dt + \delta K$ . The budget constraint thus can be written as

$$\dot{K} = wHu + rK - C - \delta K, \quad (3.16)$$

where both physical capital and efficiency labour receive their marginal product:

$$w = (1 - \alpha)Y/(uH), \quad \text{and} \quad r = \alpha Y/K. \quad (3.17)$$

The problem the representative household faces is to maximize (3.1) subject to the restrictions (3.4) and (3.16) where  $K(0)$  and  $H(0)$  are given and the technology of aggregate production is (3.15). Moreover, since every household makes the same decision, consistency requires, after optimization, that

$$\bar{H} = H. \quad (3.18)$$

The matrix of coefficient of the system of growth differential equation, (3.6) in this case is equal to (see appendix B.2)

$$M^e = \begin{bmatrix} \alpha - 1 & 0 & -B\gamma/\alpha \\ \alpha/\sigma - 1 & 1 & 0 \\ 0 & -1 & B(\alpha - \gamma)/\alpha \end{bmatrix},$$

where  $e$  stands for competitive equilibrium. This, in the absence of externality, i.e. if  $\gamma = 0$ , reduces to  $M$  in (3.6).

In comparison with our previous results in sections 3.3 and 3.4, one can see that  $M^e$  is no longer triangular and the dynamics of the output-capital ratio is not self contained. One cannot, therefore, work out the dynamics of the output-capital ratio and consumption-capital ratio using the phase diagram in the  $(z, \chi)$  plane. The off-balanced dynamics of consumption and physical capital are no



longer tractable. Moreover, since the dynamics of output growth depends on the amount of work effort, one cannot classify the local dynamics of output growth in this case either.

### 3.6.2 The socially optimal path

The optimal path from the social point of view is the solution adopted by a benevolent planner who takes all relevant information, including externality, into account. Handling the necessary conditions in a way analogous to that of the case of the decentralized economy, gives a triangular matrix of coefficient for system of growth differential equation (3.6) similar to  $M$ , except  $m_{33}$  is now replaced by  $B + B\gamma/(1 - \alpha)$  which for  $\gamma = 0$  reduces to  $M$  itself. (see appendix B.2)

Now in contrast to the decentralized economy, the dynamics of the output-capital ratio is self contained and the rate of output growth has no component in  $u$  direction. In a similar way as section 3.3, one can now obtain the U-shaped path of consumption and physical capital. Moreover, the local dynamics of output growth can be classified in a similar fashion in the  $(z, \chi)$  plane as in section 3.4. The results are summarized in the following proposition.

**Proposition 17** *In the presence of human capital externality, our findings in sections 3.3-3.5 about the dynamics of consumption, physical capital and output growth, can only be validated in the centrally planned version of the model and are not extendable to the decentralized economy.*

During the next two sections, we offer two immediate applications of our findings in sections 3.3-3.5. We postpone the application of the results of section 3.6 to the next chapter.

### 3.7 Development failure of oil economies

Sections 3.3 and 3.4 show that a Lucas economy does not react to the relative shortage of human to physical capital in a symmetric way. An economy which is relatively affluent in terms of plant and equipment, but whose labour force is not well educated, is likely to experience a decline in the level of consumption during its adjustment period to the steady state. This, with a lag, follows with a situation where investment in physical capital does not cover the depreciation of old capital leading to the falling of the stock of physical capital too. The asymmetric macroeconomic effect may also include the dynamics of output growth if the consumption-smoothing is not strong enough.

The idea outlined here can be applied to the case of oil abundant economies where an oil windfall induces a bias in their composition of wealth in favour of physical capital. In the following, we explain how these findings can shed light on some of the puzzling facts of the oil economies and in particular their growth collapse.

We also apply the idea of growth regimes, presented in section 3.5, to the effect of windfall spending on consumption in natural resource economies. We consider the transitional dynamics of a Lucas economy, that by a sudden increase in the level of physical capital, is deviated from the steady state. We show that in the paradoxical case, in which an injection of physical capital into the economy causes less education and attainment of lower steady state, the voracity effect may take place. Besides its theoretical contribution, this may explain how deterioration of the relative scarcity of human capital due to windfall spending is consistent with a lower steady state.

### 3.7.1 Oil economies as outliers

Oil economies are usually treated as outliers in cross country growth comparisons and excluded from growth regressions; Mankiw et al.(1992). The reason for doing so is that a substantial part of their recorded GDP, being the result of the extraction of existing resources, does not represent the value added. This is why Mankiw et al. argue that “one should not expect standard growth models to account for measured GDP in ...[oil] countries.”. Temple(1995), on the same grounds, takes the growth record of oil economies as observations that are highly *nonrepresentative of standard* growth theories.<sup>7</sup>

The abnormality of growth performance of natural resource economies in general are examined, amongst all by Sachs and Warner(1995,97,99), Lane and Tornell(1996), Tornell and Lane(1999) and Auty(1998,99). In oil economies, obviously a substantial part of GDP is mainly exogenous and represents the oil cycles rather than the state of the underlying economies and their choice of policies. What is less obvious is the way in which the non-oil part of GDP reacts to the oil cycles. Our aim in this section is to analyze one of the channels of transmission from oil boom to growth rate of the non-oil GDP.

The purpose of the current exposition is not to propose a growth theory for oil economies, but to rationalize some aspects of the growth record in these economies that are not well captured in the literature. We address some puzzling facts in the growth performance of oil economies. Most countries that are relatively rich in natural resources have performed poorly in the cross country growth contest in recent decades. In addition, two other puzzling facts call for expla-

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<sup>7</sup>Even in accounting for the cross country income differences, Hall and Jones(1999) exclude the value added of the mining sector to neutralize the level effect of natural resource endowment.

nation: the slow down of growth in the post-boom era relative to its pre-boom level and the poor growth performance of mineral (in particular oil) economies within the whole category of resource-based economies.

### **3.7.2 The explanations proposed**

These phenomena need justification because, by simple economic intuition, access to extra resources provides a better opportunity for investment and should stimulate growth. Five main channels of transmission from natural resource abundance to growth failure are identified in the literature. These are briefly described as follows.

**Dynamic Dutch disease** A resource boom enhances the relative price of tradeables to nontradeables in favour of the latter and reallocates the factors of production from the tradeable to the nontradeable sector. If a growth mechanism exists in the tradeable sector, such as learning by doing in manufacturing, then Dutch disease effects reduce the scope for learning by doing, and hinder the growth; Sachs and Warner(1995, 99) and Gylfason et al. (1999).

**Overaccumulation of capital** The idea of capital surplus as a cause of growth slow down of natural resource economies is proposed by Rodriguez and Sachs(1999). Resource abundant economies in this theory failed in the growth contest because “they live beyond their means.” An economy in the vicinity of its steady state, by experiencing a high enough resource boom, overshoots its steady state. The economy hence exercises a level of consumption and capital which is not affordable in the absence of resource endowment. In the post-boom era, the economy suffers from a gradual decline in its level of consumption and capital, presenting

a negative rate of growth.

**Rent Seeking** The effect of a resource boom through its effect on the allocation of human resources is considered by Baland and Francois(2000) and Torvik(2002). They believe that natural resource economies are vulnerable to rent-seeking. A resource boom, thus, may change the allocation of talent in favour of rent-seeking. As a result, effective human capital declines and growth is hampered.

**Coordination failure** In natural resource economies where powerful pressure groups have access to the resource rents, it is likely that a more than proportional redistribution of the windfall take place if there are not institutional barriers to prevent the claimants from exercising their power for appropriation of the windfall; Lane and Tornell(1996) and Tornell and Lane(1999).

**Neglecting education** Gylfason(2001) finds that school enrolment at all levels tends to be inversely related to natural resource abundance. He believes that natural resource economies, because of confidence in the wealth from the natural resources, neglect to develop their human resources. He illustrates empirical evidence that resource economies spend less than average on education and justifies their growth slow down on this ground.<sup>8</sup>

**Our explanation** Our treatment here offers an alternative hypothesis, close to those of Rodriguez and Sachs(1999) and Gylfason(2001), but in a different theoretical framework and with different conceptual intuition. We believe that

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<sup>8</sup>Stijns(2001) on the contrary argues that mineral wealth has a positive and marked impact on human capital accumulation. The link between resource abundance and human capital accumulation, in his study, is positive, running from the former to the latter.

oil-rich countries grow more slowly because they are likely to suffer from lack of development of their human capital *relative* to their stock of plant and equipment. According to this hypothesis, an oil boom is harmful for growth so long as it deteriorates the ratio of human to physical capital in favour of the latter.

This is an immediate application of our findings in sections 3.3 and 3.4 where a high ratio of physical to human capital associates with a low rate of economic growth if the consumption-smoothing effect is not strong enough. Moreover, our hypothesis provides an explanation for a high but unsustainable level of consumption in the post-boom era.

### **3.7.3 The extent of failure**

Some oil abundant economies in the Middle East and North Africa, according to Summers and Heston(1991)'s calculations, have been at the very bottom of the growth contest since the first oil shock. The extent of the accumulated per capita output lost in the 1980s in those countries, are as follows: Iraq (67.8%), Kuwait (61.5%), Nigeria (42.0%), Saudi Arabia (70.5%), and the United Arab Emirates (64.0%).

Growth performance of oil economies, in comparison with the world distribution of rate of growth of real per capita GDP, is illustrated in figure 3.6. As seen from this figure, some oil economies lie at the very bottom of the distribution and have displayed the most disappointing records during the 1980s.

To compare the performance of mineral economies and in particular oil exporters with the whole group of natural resource economies, we consult Auty(1999) who divides the economies according to their levels of GDP into large and small ones. The results are reported in table 3.1. It can be seen from this table that,

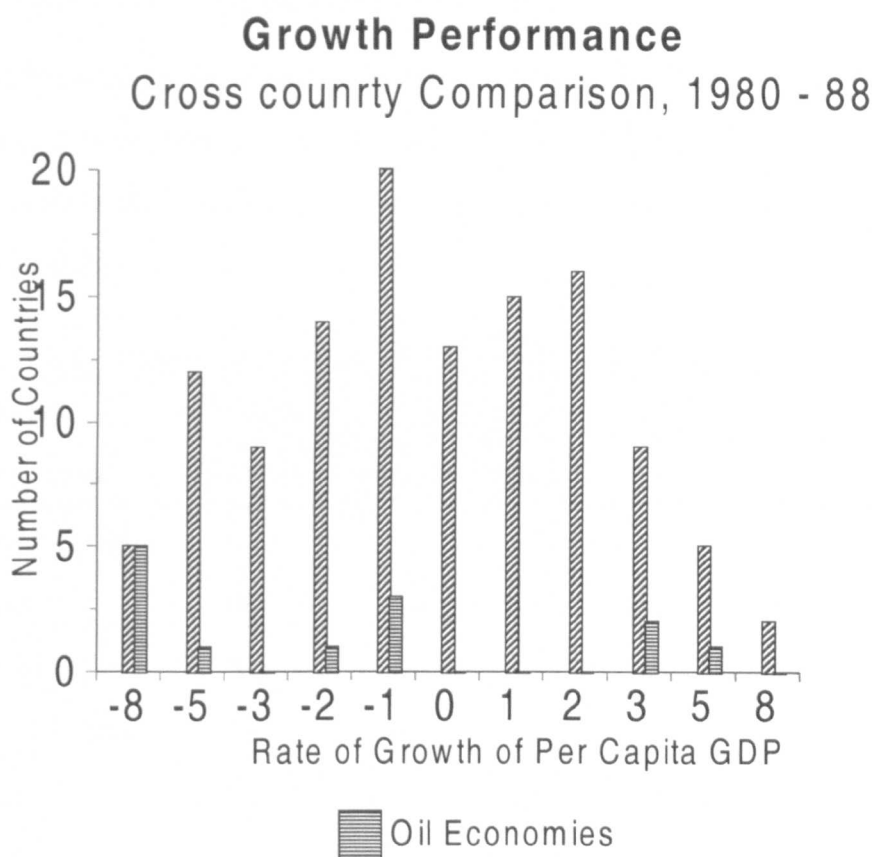


Figure 3.6: Growth performance of oil economies in comparison with the world distribution of growth rates

oil exporters, with the highest per capita income among developing countries, have substantially slowed down in the growth contest after the first oil shock of 1973.

### 3.7.4 Other regularities of oil economies

Section 2.2 captures some of the regularities of oil economies. Owing to the theme of Chapter 2 on rent-seeking, we were selective in highlighting those aspects of performances of the underlying economies that provides the motivation for the model we developed there.

Table 3.1. Resource endowment and growth record (developing countries)

<i>Resource</i>	<i>PC GDP</i>	<i>PC GDP growth (per cent per annum)</i>				
<i>Endowment</i>	<i>1970 (\$)</i>	<i>1970-3</i>	<i>1974-9</i>	<i>1980-5</i>	<i>1986-93</i>	<i>1970-93</i>
Resource poor:						
Large	196	2.8	4.9	3.1	3.5	3.7
Small	343	3.1	4.1	0.7	1.7	2.1
Resource rich:						
Large	574	0.1	-0.2	0.1	0.0	0.0
Non-mineral	250	2.0	1.7	-0.7	0.2	0.5
Hard-mineral	304	2.2	0.2	-1.8	-0.8	-0.4
Oil exporter	831	6.5	1.4	-1.7	-0.6	-1.0
All Countries	362	2.7	2.0	-0.5	0.6	0.8
Source : Auty(1999)						

Here we are concern about another aspect of the economies at hand: the extent of imbalance between human and physical capital. To have a comparable



measure of cross country distribution of physical and human capital, we consult Benhabib and Spiegel(1994) and Barro and Lee(2000) respectively. The former provides a consistent measure of physical capital for the years 1965 and 1985. In the latter study, the mean years of schooling in the total population over fifteen are reported, which is taken here as a proxy for human capital.

Armed with these data, the world distribution of physical and human capital is depicted in figures 3.7 and 3.8 respectively. What is apparent from figure 3.7 is that the distribution of physical capital among oil economies is left skewed in comparison with other countries.

Figure 3.8, reveals a different picture where the economies under question are densely located among the countries that have failed to enhance their level of human capital. Both graphs refer to 1985, when the world had experienced the first and second oil shocks, described as “one of the most remarkable international resource transfers in history” (Karl, 1997, p.25). One can conclude from these pictures that *the oil producing economies have extensively gained from physical capital deepening, but failed to develop their human capital accordingly.*

To have a comparable measure, we take the ratio of mean years of schooling to the (logarithm of) per capita stock of physical capital as an indicator of imbalance between physical and human capital<sup>9</sup>. Figure 3.9 illustrates the scatter diagram of 75 countries, whose logarithm of real per capita physical capital in 1985 exceeds 7.5 US dollars and for which data for both measures of capital were available.

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<sup>9</sup>To measure human capital across countries, Hall and Jones(1999) assume that human capital in country  $i$ ,  $H_i$  takes the form  $e^{\phi(E_i)}L_i$ , where  $L_i$  and  $E_i$  are the number of workers and their average years of schooling in country  $i$  respectively and  $\phi$  is a piecewise linear increasing function. According to this assumption, the logarithm of physical capital is a more consistent measure to be compared with the mean years of schooling.

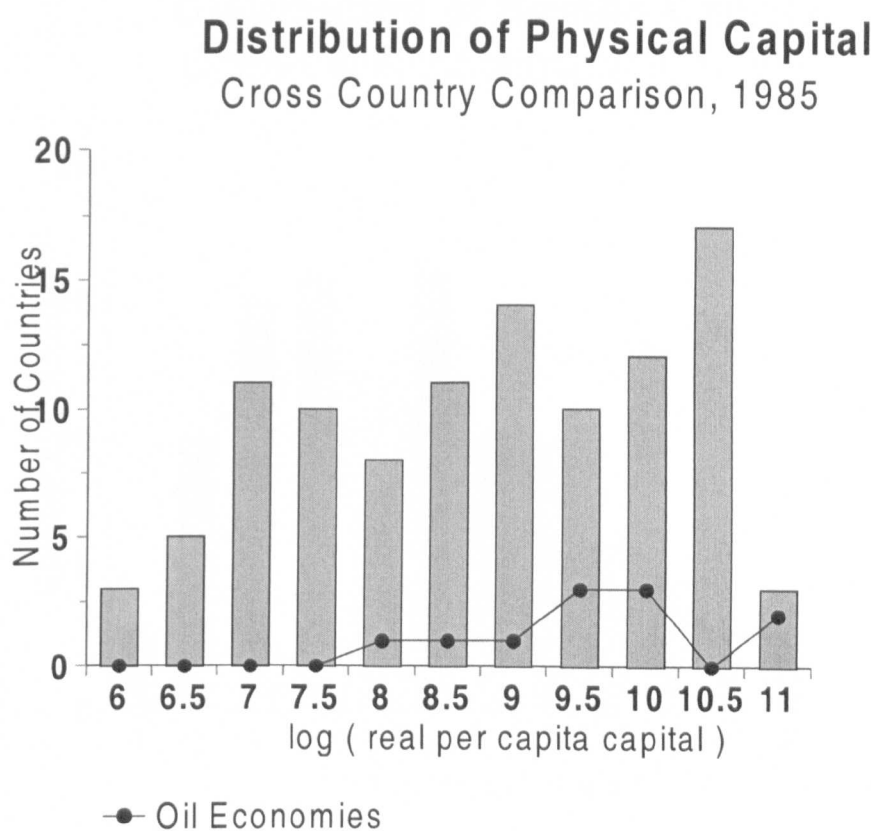


Figure 3.7: Distribution of physical capital in oil economies in comparison with the world distribution

## Distribution of Human Capital

Cross Country Comparison, 1985

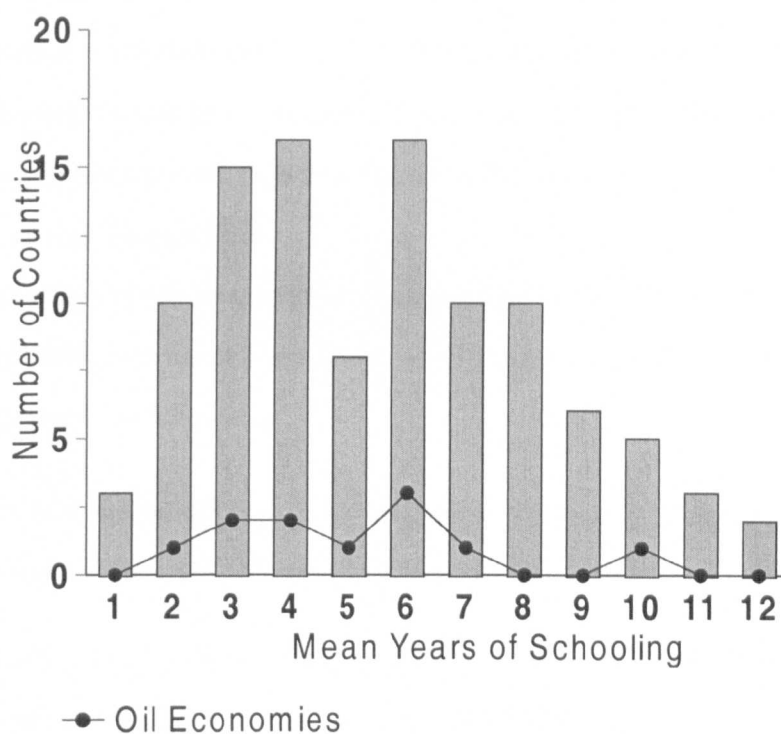


Figure 3.8: Distribution of human capital in oil economies in comparison with the world distribution

The upward sloping continuous line displays the moving average which can be interpreted as the expected level of human capital for a given level of physical capital.

As one observes from this graph, all oil economies (except Norway) are located below the average line. Furthermore most of oil abundant economies, i.e. Nigeria (NGA), Iran (IRN), Algeria (DZA), Iraq (IRQ), Saudi Arabia (SAU) and Kuwait (KWT) lie at the very bottom of the diagram. This reveals one of the regularities of oil-abundant economies that is not well covered in the literature. Given the stock of physical capital in oil abundant economies, their actual human capital is far below the expected level. We link this fact with the poor growth performance of the underlying economies.<sup>10</sup>

Some features of oil economies, documented in Gelb(1988), Karl(1997) and Amuzegar(1999), rationalize their low level of human to physical capital. They are, among many, as follows:

1. Plant and equipment can be more easily imported and implemented. Human capital accumulation on the other hand, takes time.
2. By having extra money, an economy can purchase the required hardware, but skill acquisition requires other resources too.
3. There is a tendency among oil economies to spend windfalls quickly and in an observable way. According to Gelb's investigations, the bulk of oil windfalls have been allocated in gigantic low yield projects.

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<sup>10</sup>This accords the World Bank's computations in which, among 12 main regions, the Middle East with fourth rank in terms of per capita wealth, has the lowest share of human resources and the highest share of natural capital; Auty(1998, table 4).

Referring to the imbalance between physical and human capital, the theory we are proposing is proved to be able to justify the adverse effect of an oil boom on economic growth and also a temporary boost in consumption and physical capital in the oil economies. It is also consistent with the asymmetric impact of the imbalance effect on the dynamics of the economy, high levels of capital-output ratio in the economies under question, and finally the positive income effect of oil endowment in general and oil booms in particular.

### **3.7.5 Macroeconomic effects of oil abundance**

The following regularities of oil economies have been identified so far:

1. There exists an strong association between oil abundance and the imbalance between physical and human capital.
2. Oil abundance corresponds with a high but unsustainable level of income and consumption.
3. Oil abundance is associated with a positive income but negative growth effect.

This part of study lays out an explanation for the second and third facts on the basis of the first observation. In order to do so, in the framework of a Lucas economy, we take the flow of natural resource as a catalyst for consumption and investment in physical capital. This assumption is consistent with those observations of Gelb(1988) and Gylfason(2001) that oil economies have spent the windfalls mostly on investment projects and neglected education.

In the framework of a Lucas economy described in section 3.2, we suppose that the economy stays at the steady state when it receives a windfall in terms of

# Imbalances in Stock of Capital

## Cross Country Comparison, 1985

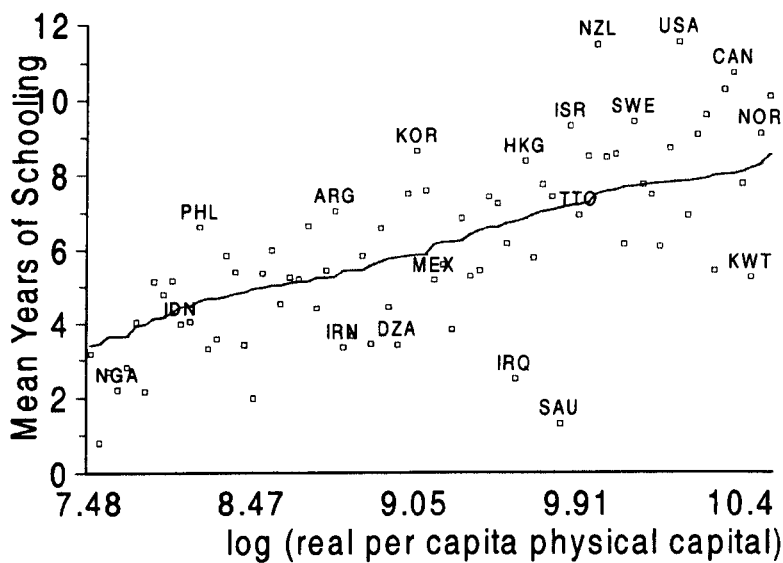


Figure 3.9: Given their stock of physical capital, oil economies poorly developed their human capital.

an oil boom. The windfall is available in units of final output which, according to Eq. (3.3), is either consumed or invested in physical capital.

Endowment of natural resources provides a new source of income for the producing economies which is expected to have both level and growth effects. To be more specific, we consider only the effect of a resource boom that is treated as an unexpected gift of nature to the economy, in GDP units, lasting only for a finite period.

As long as the boom has not occurred, the dynamics of the economy is determined along the lines specified in sections 3.3 and 3.4. When an unexpected boom occurs people realize that they have been offered a new and temporary source of wealth.

Both the magnitude and duration of the boom are assumed certain and common knowledge. In a perfect foresight environment, two sets of time profiles for the variables of the model should be identified, one in the absence of the boom and another when the boom is present. We call these, non-boom and booming situations respectively.

We treat the transitory effect of a resource boom similarly to a temporary reduction of government expenditure in the Ramsey model; Romer(2001, ch.2). A temporary injection of resources into the economy accordingly brings about a temporary boost in consumption in a static fashion. Its dynamic effect, however, acts through the intertemporal consumption decision of agents who perceive the boom as temporary. To offset the arbitrage between a high level of current consumption with a low level of future consumption, the interest rate should fall. This governs the dynamics of the economy in the post-boom era so long as the after-effects of the boom last.

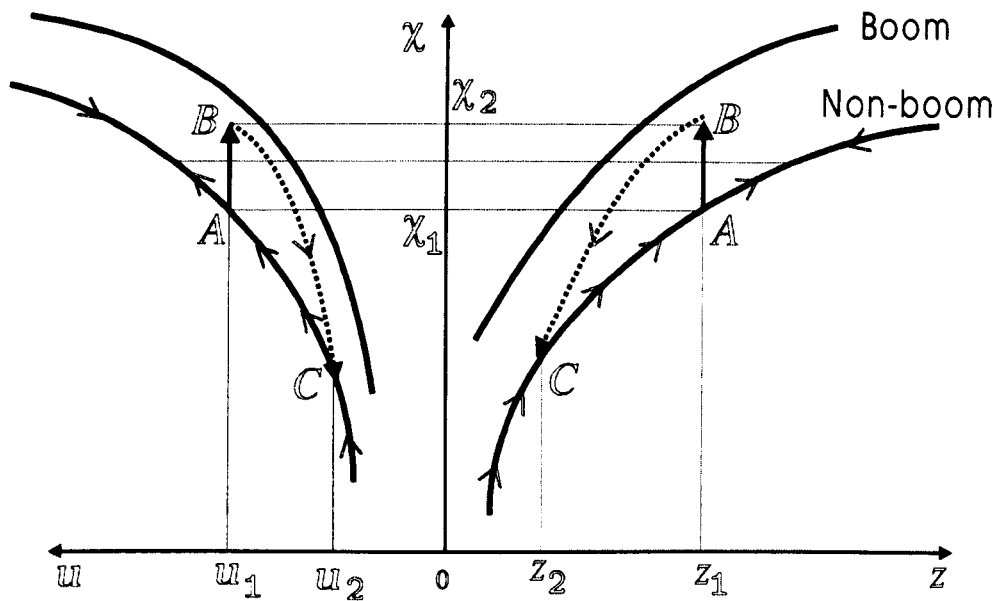


Figure 3.10: The transitional path of  $z$  and  $\chi$  in a booming economy

The transition path of the booming economy, in context of the model described in section 3.2, in  $(z, \chi, u)$  space is depicted in figure 3.10. If the windfall had lasted forever, it would have made it possible for the economy to attain a higher transitory path leading to a higher steady state. This was not however the case in our study because the resource is nonrenewable and we are only concerned about the effect of a resource boom rather than its mere endowment. Hence the higher saddle path only gives a reference for governing the divergent path of a distorted system as the result of the boom.

Suppose the economy initially stays at the steady state denoted by  $A$  in figure 3.10 with  $z = \tilde{z}$ ,  $\chi = \tilde{\chi}$  and  $u = \tilde{u}$  where an unexpected boom occurs. By bringing a new source of income with certain size and duration, the boom causes an excess supply in the product market. It partially boosts consumption and investment



in physical capital. The shorter it lasts, the stronger is its dynamic effect acting through the latter element. In particular, a permanent boom has no dynamic effect and only boosts consumption instantaneously in a one-to-one fashion. The lesser the duration of the boom in people's perception, the more they raise their savings and increase the share of their investments.

Further physical capital deepening reduces the interest rate. In a Cobb-Douglas world where the marginal and average productivity of physical capital are proportional, this is equivalent with falling of  $z$ . Along the divergent path of the booming economy  $z$ ,  $\chi$  and  $u$  fall until their path crosses the stable saddle path of the non-booming economy. This is the main mechanism through which the transitory effect of a boom operates.

This is explained through three stages. In the *first stage*, the boom has no immediate effect on  $z$  and only boosts consumption instantaneously. As a result, the economy jumps from  $A$  to  $B$  where  $z_1 = \tilde{z}$ ,  $\chi_1 > \tilde{\chi}$  and  $u_1 = \tilde{u}$ , with  $\chi_1 - \tilde{\chi}$  is increasing in both size and duration of the boom. If the boom had lasted forever, the economy would have experienced a discrete jump from non-boom saddle path to the booming one. The boom is, however, temporary and the resources are further exhaustible.

At state  $B$ , the representative consumer enjoys a high level of consumption. She realizes however that this does not last for a long time. To make her satisfied with a high current level of consumption in the face of a low level when the boom terminates, the interest rate should fall<sup>11</sup>. By a gradual decline in the interest rate, the economy moves, in the *second stage*, along the divergent path from  $B$  to  $C$  where  $z_2 < \tilde{z}$ ,  $\chi_2 < \tilde{\chi}$  and  $u_2 < \tilde{u}$ . This implies a gradual decline in

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<sup>11</sup>The stronger the willingness for consumption smoothing, i.e. the larger  $\sigma$ , the more effective is this tendency.

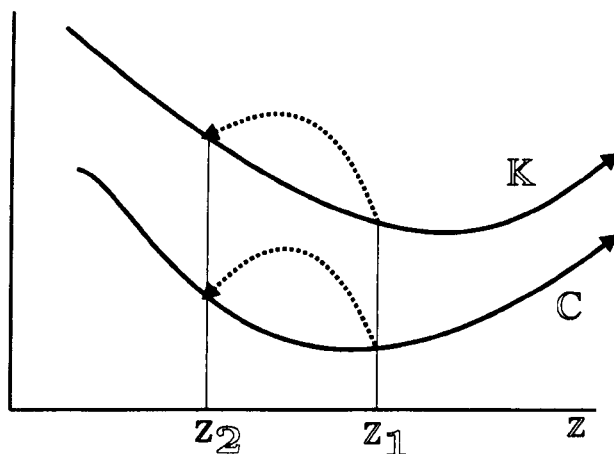


Figure 3.11: Transitory effect of an oil boom on consumption and physical capital consumption and work effort.<sup>12</sup> In this stage, one can say that the economy *lives beyond its means* because it exercises levels of consumption that, in the absence of oil rent, are not affordable.<sup>13</sup>

In the *third stage*, the economy finally moves along the non-booming saddle path toward the steady state from point *C*.

The boom has a transitory effect on consumption and physical capital. The effect is depicted in figures 3.11. Since consumption and physical capital fall and then rise during their transition path, an oil boom with a large enough magnitude or short enough duration displaces the economy from the right branch of U-shaped consumption to its left branch. This justifies a high but unsustainable boost in consumption.

To extend our explanation to the growth effect of resource booms, we rely on our findings in section 3.4. One should bear in mind that the increasing path

<sup>12</sup> Assuming  $\alpha < \sigma$ , both policy functions are increasing in  $z$ .

<sup>13</sup> This part of reasoning shares similar logic with Rodriguez and Sachs(1999).

of the output growth in the ratio of human to physical capital is valid only for low enough value of  $\sigma$ , i.e. the case where consumption-smoothing effect is not strong.

In an oil economy whose inhabitants are not patient enough, an oil boom lowers the rate of economic growth. This negative growth effect mitigates the positive income effect of boom, meaning that, in the long run “what makes [a country] poor is her wealth.” Karl(1997, p.45) The effect on output growth of boom is illustrated in figure 3.12.

### **3.7.6 Windfalls spending and the voracity effect**

The analysis that has been carried out shows that receiving a windfall, in terms of a sudden increase in the stock of physical capital, introduces a disequilibrium into the product market and affects the ratio of human to physical capital in favour of the latter. This shock, in a Lucas economy, is equivalent to a situation where the output-capital ratio,  $z$  is short of its steady state. The larger the extent of displacement, the higher is the magnitude of the boom and/or the shorter perceived its duration.

Caballé and Santos(1993, theorem 3) shows that the extent of rise in consumption, in reaction to a sudden increase in physical capital, is different in different regimes. The elasticity of consumption with respect to physical capital at the steady state is less than, greater than or equal to one in the normal, paradoxical and exogenous cases respectively.

In the normal case, the booming economy reacts to a sudden increase in the physical capital by faster accumulation of human capital. This leads the economy to a higher steady state. Consumption increases less rapidly than the size of the

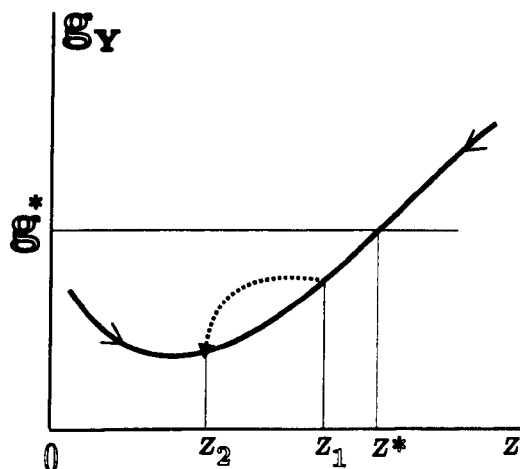


Figure 3.12: Growth effect of an oil boom along transitory path

physical capital increment in this regime.

In the exogenous case, the consumption-capital ratio is constant implying that the boost in consumption is proportional to the size of the windfall.

Finally, in the paradoxical case, people react to a physical capital shock by more consumption and less saving. The wage rate in the booming economy is high enough to discourage people from allocating more time to schooling. Consumption as a result, grows more rapidly than the size of windfall itself.

The transitory effect of a boom in the normal case was examined in subsection 3.7.5. This accommodates the case,  $\alpha < \sigma < \alpha/(1 - \alpha)$  where the consumption-smoothing effect exists but is not strong enough. In that case, the one-to-one association between the output-capital ratio and the rate of output growth around the steady state, provides a justification for the adverse growth effect of a resource boom.

We want now to extend the analysis one step further to include an economy whose inhabitants are very impatient. Along this line, we link the transitional

dynamics of the Lucas economy in the paradoxical regime, i.e. when  $\sigma < \alpha$ , to the voracity effect by which an economy, enjoying a windfall, may experience a boost in the level of consumption even higher than the size of the windfall itself.

Our treatment offers some new elements to the literature on the voracity effect. Firstly Tornell and Lane argue that when the voracity effect appears, the representative-agent models have no predictive power. To address the issue they believe that “we need more than one group to analyze redistribution.” (Tornell and Lane, 1999, p. 24)

The current study shows that the voracity effect can be addressed even without the complication that arises from interaction among multiple groups in a non-cooperative manner. To generate the voracity effect therefore we do not necessarily need to address the redistribution among agents and go beyond the representative-agent framework.

Another contribution is to bridge the voracity and consumption-smoothing effects. Although the issue has been addressed in some extent by Lane and Tornell, the link is not clear enough. In their words: “when the elasticity of intertemporal substitution is sufficiently high, ... the aggregate appropriation rate increases more than the rate of return.” (Lane and Tornell 1996, p.214)

As a necessary condition for the voracity effect to operate, they find an upper bound for  $\sigma$  which varies between half and one; (proposition 2, p. 227). We offer instead  $\alpha < 1$  as the upper bound. Moreover, we show that  $\sigma < \alpha$  gives the necessary and sufficient condition for the appearance of the voracity effect.

Our treatment of the voracity effect here, accords with Tornell and Lane(1998)’s view that “the existence of the voracity effect does not allow the smoothing effect to operate.” (p. 103) By providing a threshold for the effectiveness of the effect,

we further introduces the voracity as an extreme case of being impatient.

The existence of multiple groups who, because of the access to a common capital stock, consume excessively, is at the very heart of the Tornell-Lane models. By relying on the Caballé-Santos' result, we disjoint these two phenomena. Excessive consumption, implied merely by lack of consumption smoothing, may cause the voracity effect even if we abstract from open access and coordination failure. The effect, thus, is also reproducible in a representative-agent framework. This obviously shifts the focus from institutional side of the economy to the preferences.

Having examined the working of the model in the presence of the voracity effect, one may want to examine how far the extension of the basic Lucas model along different directions, affects the likelihood of the paradoxical regime and the appearance of the voracity effect.

Along the two lines of extension of the Lucas model in the Ladron de Guevara et al.(1997), one learns that, although the inclusion of leisure activities in the time allocation problem raises the likelihood of the paradoxical regime, the case is less likely if physical capital acts as an input to the educational sector.

Ortigueira(1998) also attempts to generalize the basic Lucas model along two lines which include the case of physical capital in the educational sector and leisure as an additional argument in the utility function. He shows that capital income taxation raises the likelihood of the paradoxical case. Alonso-Carrera(2000) also analyzes the dynamic effect of the subsidy to human capital investment. He shows that the presence of the subsidy makes the paradoxical case empirically less relevant and hence lowers the likelihood of the voracity effect.

### 3.8 Sustainability revisited

This section challenges the consistency of the two concepts of *endogenous growth* and *sustainable development*. In the light of our findings in section 3.3, we examine how, in the coexistence of steady state and transitional dynamics, the two concepts may diverge.

In practice, sustainability may be simply defined as a non-declining level of well-being across time. This can be examined according to the *flow-based* or *stock-based* measures where real per capita consumption and physical capital are two well known indicators for the former and latter respectively; Hanley(2000).

In the view of modern growth theorists, sustainability of growth is the central question to which endogenous growth theory is addressed; Aghion and Howitt(1998, ch.5). There is a debate, however, among economists about the relation between *optimality* and *sustainability*. According to Anand and Sen(1995), “there is no general presumption that sustainability will be implied by optimality in models of intertemporal allocation.”(p.5) Also “optimality and sustainability are logically distinct criteria of development. One can not be deduced from the other as a necessary consequence.”(p.36)

Dasgupta(1994) on the other hand believes that sustainable development is almost exactly what has been analyzed for decades now in the literature on optimal growth theory.

A Lucas-type endogenous growth model, as explained in section 3.2, exhibits an optimal steady state in which growth is sustainable and output, consumption, physical and human capital all grow with a positive and common rate. This achievement reconciles two logically independent concepts of optimal growth and sustainable development.

In line with Solow(1970) who believes that “the steady state is not a bad place for the theory of growth to start, but may be a dangerous place for it to end.”(p.7), we argue that the above picture is not complete as the dynamics of transition path has been overshadowed by the steady state. To understand the relation between optimality and sustainability, the transition path is as important as the steady state.

Our findings in section 3.3 show that in the Lucas model, in which the long-run growth is sustainable, under certain circumstances it is optimal for consumption and physical capital to display a negative rate of growth along the transition path. This means that although income, consumption, physical and human capital eventually grow at a common and positive rate, both consumption and physical capital might fall during their transition period implying that sustainability is violated during transition. Hence, although endogenous growth theory reconciles optimality and sustainability as two different criteria that a desirable path of development should meet, when transition is taken into account the two issues are still in conflict. This happens in a first best economy that is subject to an initial imbalance between its stock of machinery and equipment on one hand and its accumulated skills on the other hand. The main message, thus, is to point out the importance of the transition period in multi-sector endogenous growth models.

Optimal growth theory – as initiated by Ramsey – is an efficiency issue and the Maximum Principle as an established method, fulfils this task very well. Sustainability on the other hand is a matter of equity rather than efficiency. “It is possible”, Krautkraemer(1998, p.2098) argues “that a nondecreasing utility constraint forces the economy onto a path that is not Pareto-efficient.” Although



Solow(1974) in a very specific environment, shows the existence of a path that attains the highest feasible level of constant consumption and satisfies the efficiency criteria well, but obviously two concepts of efficiency and equity do not mutually coincide in the *general* framework of intertemporal allocation of resources. The former looks for an optimal path that maximizes the present value of instantaneous utility of consumption as a welfare criteria, while the latter is concerned with the non-declining path of consumption, income and physical capital.

Endogenous growth theory provides situations where the optimal growth path is sustainable. The analysis of transitional dynamics is overwhelmed, however, in the literature. This in the case where the rate of convergence is low and/or the economy is far away from its steady state is misleading. Although, thanks to endogenous growth theory, two notions were reconciled in the steady state, there exist situations where optimality criteria departs from sustainability when the optimal path of consumption and physical capital falls over time.

The falling period might also take a long time if the productivity of skill acquisition is sufficiently low and if the stock of human capital is very scarce in relative terms. This happens in the framework of an endogenous growth model where the long-run growth is sustained forever implying that consumption and physical capital eventually rise along their transition paths. The next chapter deals with this issue in more detail where another dimension of sustainability, i.e. the availability of nonrenewable resources, is added to the picture.

### **3.9 Conclusion**

The key assumption in the Lucas model, that education depends only on the pre-accumulated stock of human capital and the amount of time devoted to learning,

identifies a cluster of multi-sector endogenous growth models where only one type of capital is used across other sector(s). Within this class we explore the transitional dynamics of consumption, physical capital and output growth.

1. Dynamics of consumption and physical capital reveals the following results:

- (a) When human capital is relatively scarce, consumption and physical capital may exhibit U-shaped trends during their convergence paths toward their steady states. They firstly fall for a finite period and then rise toward their balanced growth paths.
- (b) The sequence of falling and rising of consumption and physical capital do not match. There exists situations where the rise of consumption coincides with the fall of capital and vice versa. This distinguishes the dynamics of the Lucas-type growth models from overaccumulation of capital in the Ramsey economy. Moreover, the sequence of stages in two models differ.
- (c) Rates of growth of consumption and physical capital, like the Ramsey model, move in the same direction during the transition, albeit their signs do not change at the same time.
- (d) Imposing the baseline parameters, one can observe that falling period does happen over a reasonably long period of time.
- (e) The existence of a falling stage in the transition paths of consumption and physical capital shows that in an economy that meets the sustainability condition, proposed in endogenous growth models, consumption and physical capital may fall along an optimal path violating the sustainability along the transition path.

2. We also characterize dynamics of the growth rate of output, based on the size of the intertemporal elasticity of consumption. This obtains the following results:

- (a) The U-shaped pattern of output growth, found in other studies by numerical methods, is only valid for the low and moderate values of the intertemporal elasticity of substitution. When current consumption is a good substitute for future consumption, the output growth is increasing in the ratio of physical to human capital.
- (b) For an economy whose inhabitants do not have a high degree of consumption smoothing, the relative abundance of physical capital coincides with a low rate of growth of the economy. In that situation, the idea presented here lays out an intuitive explanation of poor growth performance of economies that are affluent in terms of the physical assets but have not accumulated a sufficient level of required skills.

3. We apply our findings, on asymmetric effect of unbalanced capital in the Lucas-type growth models, to growth failure of oil economies. This gives the following results:

- (a) Oil economies, and in particular those located in the MENA region, are subject to low ratios of human to physical capital. They performed poorly in the growth contest too. We link these two observations where the former is introduced as the cause of the latter.
- (b) Besides the growth effect of the boom, which hinges on the magnitude of the intertemporal elasticity of substitution, our theory rationalizes the high but unsustainable consumption rise that oil-abundant

economies have experienced in the post-boom era. To assess the validation of our hypothesis however, one needs to estimate the magnitude of intertemporal elasticity of substitution in the underlying economies.

- (c) Another task for further research is to produce time profiles of the key variables, by numerical simulation, in the post boom era. The method carried out here works well in the phase diagram, but to compare the model's predictions with real data, one needs the time span of key variables.
- (d) One shortcoming of the current research is that it cannot analyze exactly how the size and duration of a boom shapes its after-effects. The intuition predicts that the longer the boom, the weaker is its dynamic effect which acts through the interest rate. In other words, the shorter the duration of the boom, the steeper is the path along which the interest rate is expected to decline in the second stage.
- (e) Another caveat of the current model is that it is lacking a sound justification for the poor performance of oil economies in the education sector. In a first best economy, as is the case here, this may only be justified on the grounds of low productivity of human capital. Since this is not the case in our model where the ratio of human to physical capital is low, one needs to propose an alternative hypothesis. In an open economy for instance the skilled labour can be imported from abroad which means substituting the final product with human capital when the latter is relatively scarce. One may ask then why, if any, oil economies have not followed this way?
- (f) Oil rent has a one-to-one contribution to GDP in oil countries. To

evaluate the effect of oil booms on the rate of growth, one should look at the data on the non-oil GDP rather than the GDP itself. This will enable us to properly distinguish the income effect of the boom from its growth effect. Time series on the output growth in oil economies since 1970 provides a more reliable reference by which to consider the growth effect of the boom and in particular its dynamics which are spread through time.

4. These findings are applicable to multi-sector endogenous growth models that share the Lucas assumption. This includes R&D-based growth models so long as only one type of capital is used across both sectors. Established R&D-based endogenous growth models in both horizontal and vertical innovation versions meet this criteria. This includes the expanding variety model of Romer and the creative destruction model of Aghion and Howitt where the stock of human capital can be replaced by the variety of brands and the stock of social knowledge respectively. In addition, the fraction of effort devoted to education can be replaced by the amount of research employment in both models. In the case of the horizontal innovation, Arnold(2000, p.220) finds that “the dynamic analysis [of] the Uzawa-Lucas model of growth through human-capital accumulation...applies to the optimal-growth problem in Romer’s model as well.” Next chapter shows that our findings are also applicable to the vertical innovation models.

## Chapter 4

# Nonrenewable Natural Resources and R&D-based Growth

*No economy is ever in continuous steady state equilibrium.*

Turnovsky(2000, p.487)

*Ultimately all commodities and services can be traced to natural resources.*

Dasgupta(1989, p.197)

### 4.1 Introduction

Besides its effect on industrial economies, the first oil shock initiated an intellectual challenge toward energy and exhaustible resources. It called economists'

attention to deal with the finite stock of exhaustible resources as a binding constraint on worldwide economic growth. The studies carried out by the Club of Rome with a pessimistic approach was alarming at the same time that worldwide economic growth is constrained by the scarcity of natural resources.

In spite of this Malthusian view, the majority of economists, however, argued that technical progress and capital substitution could compensate the declining trend of the essential and nonrenewable resources in the aggregate production function. In a neo-classical growth framework they considered the rate of utilization of natural resources as a factor of production. The results were dramatically in contrast with those of the Club of Rome. In their view the world could overcome the finite supply of exhaustible resources if there were a continuous flow of technical progress or if the share of physical capital in aggregate production function was larger than the share of natural resources; Stiglitz(1974), Solow(1974), Dasgupta(1974), and Dasgupta and Heal(1974,1979).

Even without technical progress, growth would be sustained indefinitely through capital accumulation alone if the asymptotic elasticity of substitution between physical capital and the rate of resource depletion exceeded one, or was equal to one, but the elasticity of output with respect to capital exceeded that with respect to resource depletion. Hence the world survives and technical progress or capital substitution can compensate the natural resource scarcity<sup>1</sup>.

Despite studies carried out during the 1980s on the macroeconomic effects of resource scarcity, by the 1990s Stiglitz's findings were still state-of-the-art in the area of growth constrained by natural resources. In his model, the main source for overcoming the natural resource scarcity was the exogenous -and hence

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<sup>1</sup>The contribution of the neoclassical growth constrained by exhaustible resource has been surveyed by Toman et al. (1995), Beltratti(1997), Krautkraemer(1998) and Neumayer(2000).

unexplained- technical progress factor.

The wave of endogenous growth theory affected the Stiglitz model too, though until recently the endogenous growth literature has not been concerned about the contribution of natural resources.<sup>2</sup> Barbier(1999) reconciles the Stiglitz model of constrained growth and Romer's model of endogenous growth to study the role of innovation in overcoming natural resource scarcity. He also considers the possibility that in low income natural resources abundant economies, the supply of innovation may be adversely affected by the rate of resource utilization.

Scholz and Ziemes(1999) demonstrate that, in the decentralized version of the expanding varieties growth model, indeterminacy of equilibrium trajectories arises when Romer's model is extended to incorporate exhaustible resources. Two types of inefficiencies in their paper are responsible for this result: monopolistic competition and information spillover.<sup>3</sup> Schou, as reported by Scholz and Ziemes(1999), also shows that, compared to the social optimum, in the decentralized version of the model the resource extraction rate could be too low or too high.

Aghion and Howitt(1998, ch.5) take environmental considerations and exhaustible resources scarcity as two different aspects of sustainability. They point out that sustainability in the latter version can be attained with weaker assumptions. They also show that the AK model, due to lack of distinction between physical and intellectual capital, cannot deal with the sustainability issue well. They propose the Shumpeterian approach of creative destruction to endogenous

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<sup>2</sup> Kolstad(2000) offers a comprehensive survey on the impact of the literature in exhaustible resources and the related policy issues.

<sup>3</sup> Poorly defined property rights of the commonly accessible natural resources is another source of inefficiency which is not taken into account in these models.



growth as an appropriate way to analyze both environmental considerations and natural resource scarcity.

Grimaud(1998) and Grimaud and Rouge(2000) illustrate how the optimal growth paths in the Aghion-Howitt model can be implemented in a decentralized economy. They also study the suboptimality of the market equilibrium with respect to the central planner's version of the model and, moreover, analyze the effects of government intervention on the characteristics of the economy, including the rate of growth and the rate of resource use in the long run.

The purpose of this Chapter is to study the transitional dynamics of the model of Aghion and Howitt in the presence of exhaustible resources introduced in Chapter 5 of their book, *Endogenous Growth Theory*. They focus on the steady state and show that continuing innovation ensures that an economy overcomes natural resource scarcity and provides a permanent level of positive consumption.

In Aghion-Howitt's model, the level of initial physical capital is considered as a *free-like* variable, implying that an economy can jump to the steady state instantaneously. In this Chapter the physical capital instead is taken as *given* at the time of planning, implying that transitional dynamics is unavoidable. We thus extend the Aghion-Howitt model to study its transitional dynamics.

The distinction between sustainability at the steady state and on the transition path is one of the main messages of the current Chapter. Moreover we derive conditions for the feasibility of the steady state and compute the rate of convergence of the economy towards it. A closed form for the fundamental variables of the economy in the long run is also derived. In addition, we provide a stability analysis of the steady state.

The Chapter is organized as follows. We firstly introduce the model in section

4.2. In comparison with the model presented in section 3.3, we provide an intuition for the new element of the model in section 4.3. The optimal growth path and steady state are consequently derived in sections 4.4 and 4.5 respectively. We analyze the dynamics of the model, including the stability analysis and the study of transitional dynamics, in section 4.6. This obtains the most important results of the Chapter. We conclude in section 4.7.

## 4.2 The model

Our presentation of the structure of the model in this section follows Aghion and Howitt (1998, ch. 3 and 5) where readers can find the missing details of the arguments. To facilitate comparison we also use their notation.

### 4.2.1 Labour force

Consider an economy populated by a fixed continuous mass of homogenous households, each endowed with one unit of skilled labour who supplies it inelastically. The population size is normalized to unity and so one is also equal to the aggregate flow of labour supply. This means that we abstract from the size effect and the variables in the model are in per capita terms.

There are two types of activity for the labour force: working in the final product market or doing research. We denote the number of people producing final product and doing research with  $L$  and  $n$  respectively implying that in each period

$$n_t + L_t = 1.$$

### 4.2.2 The resource sector

We extend the conventional growth model of creative destruction to incorporate the use of natural resources by including  $R$ , the rate of extraction of an exhaustible resource  $S$ , as an additional input in the production of final good. The relation between  $R$  and  $S$  is described as

$$S_t = S_0 - \int_0^t R_\tau d\tau, \quad (4.1)$$

where  $S_t$  ( for  $t > 0$ ) is the stock of resource at  $t$ . We assume that  $S_0$  is given and its amount is known with certainty. Moreover, for convenience, we assume that there is one pool of resources and we do not consider common utilization of the resource and its externality effect.

### 4.2.3 The final product

The economy produces one final product, denoted by  $Y$ , and a continuum of intermediate (capital) goods indexed on the interval  $[0, B]$  and denoted by  $x_i$  ( $0 \leq i \leq B$ ) where  $B$  stands for the number of available varieties of circulating capital, which represents the level of knowledge in the economy too. Each intermediate good is produced from physical capital and each can be used to produce the final good, independently of other intermediate goods, with no complementarities between them.

More specifically, the flow of final good that can be produced using intermediate good  $i$  depends only on the flow  $x_i$  itself according to the production function

$$Y_i = x_i^\alpha L^\beta R^\nu,$$

where  $L$  and  $R$  as introduced before, are the total amount of labour working in the final-product section and the rate of extraction of the natural resource respectively.<sup>4</sup> The aggregate output of the final good is therefore

$$Y = L^\beta R^\nu \int_0^B x_i^\alpha di, \quad (4.2)$$

where  $\alpha, \beta, \nu \in (0, 1)$  represent the share of intermediate good, labour and natural resource in the aggregate product respectively, and  $\alpha + \beta + \nu = 1$ .

Suppose now that there is a stock of physical capital  $K$ , which is produced along with consumption goods  $C$ , according to the production function (4.2), where the factors of production are employed in two activities of producing consumption goods and physical capital. Then:

$$Y = C + \dot{K}, \quad (4.3)$$

and there is no depreciation.

#### 4.2.4 The R&D sector

The engine of growth in the model is technical progress in the R&D sector that leads to the accumulation of knowledge and the expansion of the number of brands,  $B$ . The production of new technology uses only the amount of effort devoted to research,  $n$  and the level of social knowledge as input. So:

$$\dot{B} = \sigma \eta n B, \quad (4.4)$$

where  $\eta > 0$  is the productivity of research effort and  $\sigma > 1$  is the factor of proportionality of the level of knowledge after each innovation.

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<sup>4</sup>We deliberately abstract from the debate concerning whether man-made and natural capitals are complementary or substitutable and rely on the latter. See Solow(1987) and Kratukamer(1998) on this.

The above functional form, initiated firstly by Lucas(1988), is crucial in this model. In the two-sector endogenous growth model, this form indicates that the R&D sector does not use any final product and hence physical capital; Barro and Sala-i-Martin(1995, sec.5.2). Eq. (4.4) in this model specifies that the R&D sector does not consume any natural resource either and hence its activities is not limited by the constraints implied by resource exhaustion. Aghion and Howitt(1998, ch.5) highlight this specification as one of the advantages of the Shumpeterian approach to the AK model where the former “takes into account that the accumulation of intellectual capital is greener (in this case, less resource intensive) than the accumulation of tangible capital.”(p.162)

#### 4.2.5 The intermediate sector

The only input into the production of intermediate goods is physical capital. Considering symmetric equilibrium, the common supply of intermediate goods in each brand is defined by

$$x_i = x = K/B \quad \forall i \in [0, B], \quad (4.5)$$

indicating that the more advanced technology (larger  $B$ ) is more capital intensive. According to this specification, the total supply of intermediates at each time is equal to the stock of physical capital.

Substituting the common supply of intermediate goods from (4.5) into (4.2) yields the aggregate production function

$$Y = Bx^\alpha L^\beta R^\nu = B^{1-\alpha} K^\alpha L^\beta R^\nu. \quad (4.6)$$

This functional form is consistent with the theory of exhaustible resources where in a CES world only the Cobb-Douglas form exhibits realistic characteris-

tics consistent with the feasibility conditions implied by the scarcity of resources; Dasgupta and Heal(1974).

What we have just described is a three-sector growth model where the level of output is determined by the stock of capital  $K$ , the level of knowledge which is proportional to  $B$  and the rate of utilization of resources  $R$ , which depends on  $S$ . We refer throughout this Chapter to these three types of capital as physical (or tangible), intellectual and natural.

### 4.3 Welfare cost of running out of minerals

To make the current model comparable with that of Chapter 3, let us in the centrally planned version of the model rewrite the RHS of Eq. (3.15) as

$$H^{\beta+\gamma} K^{\alpha} u^{\beta}.$$

In comparison with the above expression, there is a new element in Eq. (4.6),  $R^{\nu}$  which we want to conceptualize in this section.

For this purpose, we take the Weitzman(1999) model as a benchmark where in a fairly general setting with plausible conditions, he derives the foregone consumption resulting from exhaustion of minerals on the basis of the time trend of mineral rents. We apply his logic to a more specific framework and find that the elasticity of production with respect to the minerals can measure this welfare indicator properly.

#### 4.3.1 The benchmark model

Weitzman analyses the welfare cost of running out of exhaustible resources, in terms of the implied limits to worldwide growth of future consumption. By

considering a linear welfare measure of the form

$$U = \int_0^{\infty} C(t)e^{-rt}dt,$$

he compares the present discounted value of consumption in two scenarios, namely the *current* situation where the worldwide growth is limited by a declining trend of exhaustible resources, and a *hypothetical* scenario in which the current flow of exhaustible resources would be available forever and the world would never run out of them. He then proposes

$$\frac{\Delta U}{U} = \frac{U^h - U^a}{U^a},$$

as the required welfare loss where  $U^a$  and  $U^h$  refer to the solution of the optimal growth problem in the actual and hypothetical cases respectively.

In a clever way, Weitzman concludes then

$$\frac{\Delta U}{U} = \frac{P^a(0).R(0)}{C^a(0) + I^a(0)} \quad (4.7)$$

where  $P$  is the net price (out of the marginal cost) of the minerals,  $R$  refers to its rate of extraction, and  $C$  and  $I$  denote the level of consumption and investment (in the aggregate capital) respectively. In addition, all of the variables refer to the optimal solution of the real growth problem limited by a depletable flow of exhaustible resources.

Although the main point of his paper lies in the empirical side of the story, this Chapter is concerned only with the heuristic approach taken in its theoretical side. Instead being faced with “a computable general equilibrium model having dynamic specifications of oil demand and supply response functions, a sophisticated treatment of expectation, knowledge of world oil reserves and exploration costs, estimates of elasticities of substitution between oil and all other relevant

factors of production, sectorial projections of technological progress, learning curves, macroeconomic growth forecasts" (p.691), Weitzman simplifies the problem using the fact that the prices of minerals indicate the market judgement of the overall welfare loss from running out of them.

### 4.3.2 The CES technology

We claim that Weitzman's theoretical contribution might be even more applicable than he has found to be so, if one confines the analysis to the CES world. Consider the worldwide technology of production as

$$Y = F(K, R),$$

where  $Y$  represents the aggregate final output, and  $K$  and  $R$  refer to the broadly defined capital and flow of mineral respectively. In Weitzman's analysis  $F$  could take virtually any form.

Dasgupta and Heal(1974) argue that in a CES world where the elasticities of substitution between natural resources and the aggregate capital is constant, only the Cobb-Douglas form exhibits realistic characteristics which are consistent with the feasibility conditions implied by the scarcity of resources. It is reasonable therefore to limit our attention in a CES economy, to the production function of the form,

$$Y = AK^{1-\nu}R^{\nu}.$$

Then, in line with Weitzman, we take the final product market as perfectly competitive to conclude

$$\nu = \frac{P^a(0).R(0)}{C^a(0) + I^a(0)}, \quad (4.8)$$



which is the result of combining Eq. (4) and (8) of Weitzman in the Cobb-Douglas context with constant returns to the reproducible capital and the flow of nonrenewable resources. Now plugging the left hand side of Eq. (4.8) into (4.7), gives  $\nu$  as the welfare cost of resource exhaustion.

## 4.4 Optimal growth

Now suppose that the representative agent's welfare is given by the intertemporal isoelastic utility function

$$W = \int_0^\infty \frac{C^{1-\epsilon} - 1}{1-\epsilon} e^{-\rho t} dt, \quad (4.9)$$

where  $C$  represents consumption and  $\rho > 0$  and  $1/\epsilon > 0$  denote the rate of time preference and intertemporal elasticity of substitution respectively.

The problem of optimal growth is that of choosing the rates of consumption  $C$ , research employment  $n$ , and extraction of resource  $R$ , at each date so as to maximize  $W$  subject to (4.4) and

$$\dot{S} = -R, \quad (4.10)$$

$$\dot{K} = B^{1-\alpha} K^\alpha (1-n)^\beta R^\nu - C, \quad (4.11)$$

where  $B_0$ ,  $K_0$  and  $S_0$  are given at the time of planning.

The Hamiltonian of the optimization problem is

$$H = \frac{C^{1-\epsilon} - 1}{1-\epsilon} + \lambda [B^{1-\alpha} K^\alpha (1-n)^\beta R^\nu - C] + \mu \sigma \eta n B - \xi R.$$

There are three state variables ( $K$ ,  $B$  and  $S$ ), three costate variables ( $\lambda$ ,  $\mu$  and  $\xi$ ), and three control variables ( $C$ ,  $n$  and  $R$ ). Assuming an interior solution, the

first order (static efficiency) conditions are

$$C^{-\epsilon} = \lambda, \quad (4.12)$$

$$\beta Y / [(1 - n)\sigma\eta B] = \mu/\lambda, \quad (4.13)$$

$$\nu Y / R = \xi/\lambda. \quad (4.14)$$

The Euler equations (dynamic efficiency conditions) are

$$g_\lambda = \rho - \alpha Y / K, \quad (4.15)$$

$$g_\mu = \rho - (\lambda/\mu)(1 - \alpha)(Y/B) - \eta\sigma n, \quad (4.16)$$

$$g_\xi = \rho, \quad (4.17)$$

where for each variable like  $z$ , we denote its exponential rate of growth with  $g_z = \dot{z}/z$ .

The transversality conditions are

$$\begin{aligned} \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t K_t &= 0, \\ \lim_{t \rightarrow \infty} e^{-\rho t} \mu_t B_t &= 0, \\ \lim_{t \rightarrow \infty} e^{-\rho t} \xi_t S_t &= 0. \end{aligned} \quad (4.18)$$

Before returning to the algebra, we follow Barbier(1999) to build intuition about these conditions. Eq. (4.12) describes the optimality rule for consumption. It indicates that, along the optimal path, the marginal utility of consumption must equal the shadow price of accumulated capital. Condition (4.13) determines the optimal amount of research effort. It shows that the marginal productivity

of the labour force in the final product relative to the marginal contribution of research in innovation, must equal the relative price of intellectual capital to the tangible one. Condition (4.14) indicates that the marginal productivity of resource inputs must equal the relative price of the resource stock to capital. Condition (4.15) shows that the percentage change in the shadow price of physical capital must equal the discount factor less the marginal productivity of capital. Similarly, condition (4.16) shows that the percentage rate of change in shadow value of intellectual capital should equal the discount factor less the rate of growth of social knowledge and the relative price of physical capital to intellectual one times the marginal productivity of social knowledge. Finally, condition (4.17) indicates that the capital gains of holding on the resource stock on the optimal path must be equal to its opportunity costs.

In addition, the transversality condition corresponding to  $S$ , using (4.17) gives  $S_t \rightarrow 0$ , which due to the finiteness of  $S$  and nonnegativity of  $R$ , implies

$$\lim_{t \rightarrow \infty} R_t = 0,$$

meaning that *along an optimal growth path, the entire stock of resource is exhausted and the rate of utilization of resource also tends to zero.*

From (4.12) and (4.15), one obtains the familiar *Euler equation for consumption* in this economy

$$g_C(t) = [\alpha(B_t/K_t)^{1-\alpha}(1 - n_t)^\beta R^\nu - \rho]/\epsilon,$$

indicating that *in the absence of population growth, an economy with exhaustible resources is sustainable if increases in the ratio of intellectual capital to the tangible capital compensates for the finiteness of the essential resources.*

By simple manipulation, we derive the optimal rate of growth of the variables

that we are concerned with in the following proposition.

**Proposition 18** *The optimal rates of growth of the main variables of the model are expressed as follows:*

$$g_S = -R/S, \quad (4.19)$$

$$g_B = \sigma\eta n, \quad (4.20)$$

$$g_C = [\alpha(Y/K) - \rho] / \epsilon, \quad (4.21)$$

$$g_K = (Y - C)/K, \quad (4.22)$$

$$g_n = -(1 - \alpha)\sigma\eta\left(\frac{1 - n}{\beta} + \frac{1}{\alpha}\right) + \frac{C}{K}, \quad (4.23)$$

$$g_Y = \left(\frac{1 - \alpha}{\alpha}\right)\sigma\eta - \frac{C}{K} + \alpha\frac{Y}{K}, \quad (4.24)$$

$$g_R = \frac{(1 - \alpha)\sigma\eta}{\alpha} - \frac{C}{K}. \quad (4.25)$$

Comparing Eq. (4.24) and (4.25) suggests that growth in resource use is determined by the rate of growth of output less the marginal productivity of capital. This can be written as

$$\alpha Y/K = g_Y - g_R,$$

which addresses equality between rates of return in the production of physical capital and the extraction of exhaustible resources, known as *the Hotelling condition*.

Eqs. (4.19)-(4.25) define the optimal growth path of the main variables of the model. As one considers, the optimal growth path of all variables are linear combinations of  $Y/K$ ,  $C/K$ ,  $n$  and  $R/S$ . These are called *fundamentals* of the model.

## 4.5 Steady state analysis

### 4.5.1 Specification

Definition of the steady state requires that  $C$ ,  $K$  and  $Y$  grow at a constant common rate denoted by

$$g^* \equiv g_y^* = g_C^* = g_K^*, \quad (4.26)$$

where  $*$  stands for the steady state. This implies that capital productivity remains constant at the steady state.

Moreover, the research effort and the proportion of the labour force working in the product market remains constant, which requires

$$g_n^* = -g_L^* = 0. \quad (4.27)$$

Furthermore following Barbier(1999), in a growth model constrained by a finite supply of natural resources and to investigate whether natural resource scarcity operates as a binding constraint on the economic growth, it is worth exploring the condition under which the long run equilibrium is characterized by the rate of resource utilization  $R/S$ , converging to a steady state value  $(R/S)^*$ . Thus an additional steady state condition imposed on the optimal path of the economy is

$$g_{R/S}^* = 0. \quad (4.28)$$

Equations (4.26 - 4.28) define the balanced growth path(BGP) at the steady state. In the following, firstly we impose the steady state conditions on the optimal growth equations to derive the BGP. We then examine the existence and uniqueness of the steady state and finally solve the equations governing the BGP path to derive the long run levels of the fundamentals.

### 4.5.2 The balanced growth path

We impose the constraints determining the steady state, i.e. conditions (4.26 - 4.28) on the equations which define the optimal growth path derived in section 4.4. Imposing (4.28) on (4.25) and (4.19) gives the steady state rate of utilization of the resource as

$$\left(\frac{R}{S}\right)^* = \left(\frac{C}{K}\right)^* - \frac{(1-\alpha)\sigma\eta}{\alpha}. \quad (4.29)$$

Now considering (4.19) and (4.28), this gives

$$g_R^* = g_S^* = \left(\frac{1-\alpha}{\alpha}\right)\sigma\eta - \left(\frac{C}{K}\right)^*, \quad (4.30)$$

which means that in the steady state, the stock of resources and its rate of utilization both decline with a common rate which is inversely related to the consumption-capital ratio.

Imposing now, after log-differentiation from (4.6), conditions (4.26) and (4.27) and substitutes from (4.30), one obtains

$$g_B^* = g^* + \frac{\nu}{1-\alpha} \left(\frac{R}{S}\right)^*. \quad (4.31)$$

This, considering  $(R/S)^* \geq 0$ , means that *in the presence of an exhaustible resource, the balanced rate of growth is lower than the rate of knowledge accumulation.*<sup>5</sup> *The higher rate of utilization of the resource or the higher resource contribution in the final product, the lower is the balanced rate of growth.*

This is a justification of the argument that natural resource intensity is harmful for growth<sup>6</sup>. Using a Cobb-Douglas production function with a declining flow

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<sup>5</sup>Log-linearization from (4.6), at the steady state gives  $g_R^* = (g^* - g_B^*)/\nu$  which implies  $R_t = R_0 \exp[(g^* - g_B^*)t/\nu]$ . The integral concerning resource constraint,  $\int_0^\infty R_t dt \leq S_0$  therefore converges if and only if  $g^* < g_B^*$ .

<sup>6</sup>This is entirely different from what has been presented in section 3.7. Whereas here we

of natural resource as an essential factor of production, makes the growth slower with respect to the no-resource case, i.e. when  $\nu = 0$ . In the latter case the balanced rate of growth is equal to the rate of knowledge accumulation.

By log-differentiation from (4.6) and (4.14) and substituting  $g_\lambda$  and  $g_R$ , one derives the optimal balanced rate of growth of the economy as an increasing function of the research effort

$$g^* = g(n^*) = \frac{(1 - \alpha)\sigma\eta n^* - \rho\nu}{(1 - \alpha) + (\epsilon - 1)\nu}, \quad (4.32)$$

meaning that research effort has a positive growth effect which is consistent with the conventional creative destruction model.<sup>7</sup> In that model, however, the balanced growth rate is equal to the growth rate of intellectual capital which, according to (4.4), is equal to  $n^*$ , the steady state level of research employment.

In the conventional model of creative destruction in the steady state output, consumption, physical and intellectual capital grow at a common rate  $g^* = \sigma\eta n^*$  (see section 3.2.2 of Aghion and Howitt). What we found here is that incorporating the natural resource into the model not only causes the share of natural resource,  $\nu$ , to come into account but in this case the elasticity of substitution of consumers also matters. The higher the degree of consumption-smoothing, the flatter is  $g^*(n)$  and the less is the growth effect of research activity.

The relation between the balanced growth rate,  $g^*$  and the level of research effort,  $n$  is depicted in figure 4.1. As we consider, in the presence of exhaustible address the effect of using exhaustible natural resources on growth, there we refer to the *endowment* of natural resources.

<sup>7</sup>Research employment raises the innovation and hence enhances growth. The cost of allocation of labour, however, is the contraction of the volume of the final output.

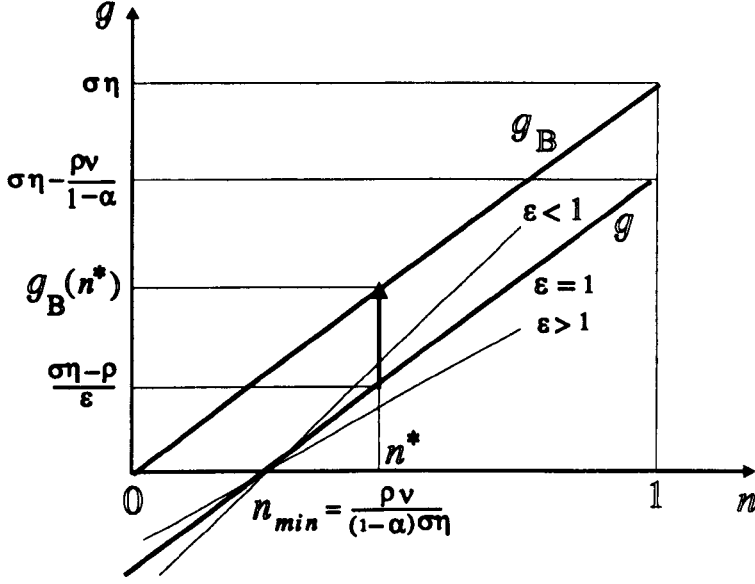


Figure 4.1: In the presence of exhaustible resources, a minimum level of research effort is required to keep the economy growing.

resource a minimum level of research effort, namely

$$n_{\min} = \frac{\rho\nu}{(1-\alpha)\sigma\eta} > 0 \quad (4.33)$$

is required to compensate the declining trend of resource use and provide a positive rate of growth. Obviously for  $\nu = 0$  we have  $n_{\min} = 0$ .

Imposing (4.26) on (4.21), (4.22) and (4.24) gives two independent linear equations in  $Y/K$  and  $C/K$ . Using algebra, we derive the steady state level of the output-capital ratio and the consumption-capital ratio from these equations as

$$\left(\frac{Y}{K}\right)^* = \frac{\sigma\eta}{\alpha}, \quad (4.34)$$

and

$$\left(\frac{C}{K}\right)^* = \frac{\sigma\eta}{\alpha} - \frac{\sigma\eta - \rho}{\epsilon}. \quad (4.35)$$



Now we impose (4.27) on (4.23). This, after substitution from (4.35), gives the steady state level of research effort as

$$n^* = \frac{\nu}{1-\alpha} + \frac{\beta}{(1-\alpha)\sigma\eta} \left( \frac{\sigma\eta - \rho}{\epsilon} \right), \quad (4.36)$$

which is increasing in  $\nu$  and decreasing in  $\epsilon$  as expected.

By substituting the steady state level of the consumption-capital ratio from (4.35) in (4.29) we derive the rate of utilization of the resource at the steady state as

$$\left( \frac{R}{S} \right)^* = \sigma\eta - \frac{\sigma\eta - \rho}{\epsilon}, \quad (4.37)$$

which gives the optimal rate of depletion of resource at the steady state as

$$R_t = R_0 \cdot \exp \left[ \left( \frac{\sigma\eta - \rho}{\epsilon} - \sigma\eta \right) t \right].$$

In the simplest case where  $\epsilon = 1$ , the rate of decline of resource use is equal to the discount rate,  $\rho$ . This means that our model, like those of Barbier(1999), can produce Hotelling's rule as a special case. Moreover, in the absence of R&D (i.e. when  $\sigma$  and  $\eta$  are zero) we have  $g_R = -\rho/\epsilon$  which is the same as the findings of the neoclassical model of Dasgupta and Heal(1974).

For deriving the balanced rate of growth, we substitute  $(Y/K)^*$  from (4.34) into (4.21) which results in:

$$g^* = \frac{\sigma\eta - \rho}{\epsilon}. \quad (4.38)$$

One considers from the above expression that *the economy grows faster in the long run, the more productive is the R&D activities; the lower is the rate of time preference or the larger is the intertemporal elasticity of substitution*. The rate of knowledge accumulation at the steady state can be obtained by substituting

$n^*$  from (4.36) into (4.4) as

$$g_B^* = \sigma\eta - \frac{\beta}{1-\alpha} \left( \sigma\eta - \frac{\sigma\eta - \rho}{\epsilon} \right). \quad (4.39)$$

### 4.5.3 The existence and uniqueness

On the balanced growth path  $C$ ,  $K$  and  $Y$  grow at the same rate,  $n$  does not change and  $R$  and  $S$  decline at the same rate too. Thus the rate of growth of the fundamental variables  $Y/K$ ,  $C/K$ ,  $n$  and  $R/S$  on the steady state must be zero.

According to optimal growth equations, one can derive the growth rates of fundamentals of the model in a system of growth differential equations, which, due to the functional form of production function, is linear in those variables themselves:

$$\begin{bmatrix} g_{Y/K} \\ g_{C/K} \\ g_n \\ g_{R/S} \end{bmatrix} = \begin{bmatrix} \alpha - 1 & 0 & 0 & 0 \\ \alpha/\epsilon - 1 & 1 & 0 & 0 \\ 0 & 1 & -\alpha\lambda/\beta & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y/K \\ C/K \\ n \\ R/S \end{bmatrix} - \lambda \begin{bmatrix} 1 \\ \rho/(\epsilon\lambda) \\ (\nu - 1)/\beta \\ 1 \end{bmatrix}, \quad (4.40)$$

where  $\lambda = (\alpha - 1)\sigma\eta/\alpha$ .

We call the RHS of the above system  $Ax - b$ , where  $A$  is the  $4 \times 4$  matrix of coefficients,

$$x \equiv (Y/K, C/K, n, R/S)^T,$$

is the vector of fundamental variables and  $b$  is the constant vector of parameters.

The triangular structure of the matrix of coefficients, suggests that the dynamics of the model is fully tractable. Moreover, the non-zero entry in the last row of  $A$  indicates that *the rate of decline of resource utilization is affected by*

the consumption-capital ratio, while the zero entries of the last column of  $A$  indicate that the dynamics of the core of the economy is not affected by the rate of resource utilization.

According to the previous section, the steady state is defined as

$$\{x \in \mathbb{R}_+^4 : Ax = b\}.$$

Hence the existence and uniqueness of the steady state can be analyzed by characterization of the matrix of coefficient  $A$  for which we have  $\det A = -(1 - \alpha)^2 \sigma \eta / \beta < 0$ . Thus we conclude the following statement<sup>8</sup>:

**Proposition 19** *For the model economy, there exists a unique steady state which is completely described by the parameters of the model as*

$$x^* = A^{-1}b. \quad (4.41)$$

**Conclusion 9** (i) *The unique steady state is characterized by the set of known parameters as follows:*

$$\begin{aligned} (Y/K)^* &= \sigma \eta / \alpha, \\ (C/K)^* &= \sigma \eta / \alpha - (\sigma \eta - \rho) / \epsilon, \\ n^* &= \nu / (1 - \alpha) + \beta(\sigma \eta - \rho) / [(1 - \alpha)\sigma \eta \epsilon], \\ R^* &= \sigma \eta - (\sigma \eta - \rho / \epsilon). \end{aligned}$$

(ii) *The growth rates along the balanced optimal path are:*

$$\begin{aligned} g_Y^* &= g_C^* = g_K^* = (\sigma \eta - \rho) / \epsilon, \\ g_B^* &= \sigma \eta - \beta \left( \sigma \eta - \frac{\sigma \eta - \rho}{\epsilon} \right) / (1 - \alpha), \\ g_R^* &= g_S^* = \frac{\sigma \eta - \rho}{\epsilon} - \sigma \eta. \end{aligned}$$

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<sup>8</sup>Nili(2000) explains how the violation of constant returns to scale implies the nonuniqueness of the steady state and in this case what the characteristics of multiple equilibria are.

(iii) *The (unique) transversality condition is  $(\sigma\eta - \rho) / \epsilon < \sigma\eta$ .*

For  $g^*$  in (4.38) to be positive it is necessary to have  $\rho < \sigma\eta$ . In addition the transversality condition corresponding to  $K$ , in conjunction with Eqs. (4.17) and (4.34), implies  $\sigma\eta - \rho < \sigma\eta\epsilon$ . We combine these inequalities in the following condition:

**Condition 6**  $0 < \sigma\eta - \rho < \sigma\eta\epsilon$ .

We show in the following that condition 6 is the one required for the steady state to be well defined.

**Proposition 20** *Condition 6 is the necessary and sufficient condition for sustainability of the growth on the steady state ( i.e. for  $g^* > 0$  ) in the presence of an exhaustible resource which is essential for production, and also for  $W$  to be bounded. Moreover further to  $(Y/K)^*$  which based on the range of the parameters of the model is positive, condition 6 is the sufficient condition for the steady state to be well defined, i.e.  $0 < n^* < 1$ ,  $(C/K)^* > 0$  and  $(R/S)^* > 0$ .*

One may observe from condition 6 that an optimal negative growth of output is possible, if the subjective rate of discount,  $\rho$  is sufficiently large, i.e. larger than the effectiveness of the R&D sector,  $\sigma\eta$ . Obviously the condition for sustained growth should be modified in the presence of population growth or capital depreciation.

Considering Eqs. (4.5) and (4.31) we conclude the following statement:

**Proposition 21** *In the presence of an exhaustible resource (i.e. when  $\nu > 0$ ), the rate of balanced growth is less than the rate of knowledge accumulation at the steady state. This implies that, at the steady state, the total supply of intermediate*

*goods grows at the balanced rate of growth while the supply of each individual branch declines proportionately with the flow of natural resources.*

From the Cobb-Douglas form of the aggregate production function in (4.6) we know that both the intermediate goods  $x$ , and rate of extraction of the resources  $R$ , are essential for production. In (4.29) we conclude that in the steady state  $R$  is declining without bound. In the above we found the same result for  $x$ . Furthermore, the rate of decline of both variables are proportional in the steady state:

$$g_x^* = \frac{\nu}{1-\alpha} g_R^*.$$

*This indicates that integrating an exhaustible resource into the aggregate production is harmful for sustainability of the growth directly through its declining rate of extraction and indirectly via the supply of intermediate goods.*

#### **4.5.4 A numerical illustration**

To illustrate the working of the model, a practical example is given in this section. We should emphasize, however, that it is not a calibration exercise but to show how the model works. For this reason we select a set of plausible values for technology parameters  $\alpha = 0.5$ ,  $\beta = 0.45$ ,  $\nu = 0.05$ , preference parameters  $\epsilon = 2$ ,  $\rho = 0.02$  and R&D parameters  $\sigma\eta = 0.06$ .

The size of the variables of the model in this setting is described in table 4.1, where the figures are in percent (except the life of the stock of the resource which is reported in years). In addition, excluding the rate of convergence, the rest of the variables refer to their values at the steady state.

Two figures in the above table seem odd with respect to the empirical evidence: the high speed of convergence, i.e. 6 percent in contrast to the adjustment

rates of the order of 2 percent per annum e.g. in Mankiw et al. (1992), and the high level of research employment, i.e. 35 percent relative to e.g. 18.2 percent in Barro and Sala-i-Martin(1995, sec.5.2.2).

Table 4.1 Numerical illustration of the steady state

Balanced rate of growth	1.5
Output-capital ratio	12.0
consumption-capital ratio	10.5
Saving rate ( $= \alpha g^* / \sigma \eta$ )	16.7
Rate of resource utilization	4.0
Life of the resource stock ( in years)	25.0
Interest rate ( $= \sigma \eta$ )	6.0
Research employment of skilled labour	35.0
Rate of knowledge accumulation	2.1
Rate of convergence toward the steady state	6.0

Regarding the former, we should emphasis that the implausibly high rate of convergence is a common problem in the Lucas-type models where the technology of innovation depends only on the flow of research input and the existing stock of knowledge. Ortigueira and Santos(1997) show how the introduction of adjustment costs into the accumulation of physical capital, significantly reduces this figure.

Concerning the second problem, as we mentioned in Eq. (4.36), the incorporation of nonrenewable resources pushes the steady state level of research employment upwards. The contribution of exhaustible resources in the long-run research effort i.e.  $\nu / (1 - \alpha)$  here is 10 percent which justifies the high value of

$n^*$  in the above table. Moreover increasing  $\epsilon$  by one unit, decreases  $n^*$  by 10 percent.

## 4.6 Transition dynamics

The purpose of this section is to investigate the dynamic properties of the model around the steady state and its transitional path toward it. We analyze firstly the stability of the steady state. This is carried out by linearization of the dynamical system describing the optimal growth paths around the steady state. We then study the transitional path of the economy toward the steady state.

### 4.6.1 Local dynamics and stability

By linearization, the local dynamics of the model around  $x^*$  is approximated by the following system of linear differential equation

$$\frac{d}{dt}(x - x_t) = A^*(x - x^*), \quad (4.42)$$

where  $A^* = [a_{ij}^*]$  is the Jacobian matrix of the system described by (4.40) evaluated at  $x^*$ . For this matrix we have  $a_{ij}^* = a_{ij} \cdot x_i^*$  where  $a_{ij}$  refers to the entry of  $i$ -th row and  $j$ -th column of matrix  $A$ .

Since  $A^*$  like  $A$  is lower triangular one can determine its eigenvalues as

$$\lambda(A^*) = \left\{ (\alpha - 1) \left( \frac{Y}{K} \right)^*, \left( \frac{C}{K} \right)^*, \frac{(1 - \alpha)\sigma\eta}{\beta} n^*, \left( \frac{R}{S} \right)^* \right\},$$

or after substitution from Eqs. (4.34 - 4.37)

$$\lambda(A^*) = \left\{ \frac{(\alpha - 1)\sigma\eta}{\alpha}, \frac{\sigma\eta}{\alpha} - \left( \frac{\sigma\eta - \rho}{\epsilon} \right), \frac{\nu\sigma\eta}{\beta} + \frac{\sigma\eta - \rho}{\epsilon}, \sigma\eta - \frac{\sigma\eta - \rho}{\epsilon} \right\}.$$

Condition 6 implies that  $A^*$  has one negative eigenvalue  $\lambda = (\alpha - 1)\sigma\eta/\alpha$  whose magnitude determines the speed of convergence along the transition path.

There are also three positive eigenvalues:  $\lambda_2$ ,  $\lambda_2$  and  $\lambda_4$ . They are real but not necessarily distinct. There is the possibility of overshooting, but as will be described here, one can set the initial conditions in such a way that the dynamics of the system become more tractable.

The solution of the linearized system (4.42) can be written in the general form as

$$x_t - x^* = \sum_{i=1}^4 c_i \Lambda_i t^{\delta_i} \exp(\lambda_i t), \quad (4.43)$$

where  $\Lambda_i$  is the eigenvector corresponding to the eigenvalue  $\lambda_i$ ,  $\delta_i$  is a nonnegative integer less than multiplicity of  $\lambda_i$ , and  $c_i$  is the constant of integration.

Since there is only one negative eigenvalue, i.e.  $\lambda_1$ , the only subspace generated by its associated eigenvector  $\Lambda_1$  is stable. Considering the initial conditions, although  $B_0$ ,  $K_0$  and  $S_0$  are given we can freely determine  $C_0$ ,  $n_0$  and  $R_0$ . Hence we can choose  $x_0$  such that  $x_0 - x^*$  has no component in the subspace generated by  $\Lambda_i$  ( $i = 2, 3, 4$ ). This requires  $c_i = 0$  for  $i = 2, 3, 4$ .

By replacing these constraints in (4.43), the stable solution can be characterized as

$$x_t - x^* = [(Y/K)_0 - (Y/K)^*] \left[ \frac{(Y/K)_t}{(Y/K)_0} \right] \Lambda_1 \cdot \exp \left[ \left( \frac{\alpha - 1}{\alpha} \right) \sigma \eta t \right],$$

where  $\Lambda_1$  rules out unstable paths<sup>9</sup>.

The rate of convergence of the economy towards the steady state is determined solely by  $\alpha$ ,  $\sigma$  and  $\eta$  and is independent from the preference parameters<sup>10</sup>. There is a one-dimensional stable saddle path therefore converging to the steady state  $x^*$ .

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<sup>9</sup>The first component of  $\Lambda_1$  is normalized to one.

<sup>10</sup>This is not however the case if we augment the law of motion of  $B$  by the effect of physical capital on the innovation process; Ortigueira and Santos(1997).



### 4.6.2 Coexistence of the transition and the steady state

Now it is time to ask how a central planner sets the control variables at their initial positions. The control variables are required to be set such that  $x_0$  is equated with  $x^*$  where the elements of  $x^*$  are described by the RHS of Eqs. (4.34 - 4.37). Thus given  $K_0$ ,  $B_0$  and  $S_0$  and the parameters of the model, to avoid the social cost of being out of steady state, the social planner solves Eq. (4.34) at the time of planning so that

$$(B_0/K_0)^{1-\alpha} \left( \frac{\beta}{1-\alpha} \right)^\beta [1 - (\sigma\eta - \rho)/\sigma\eta\varepsilon]^\beta (\sigma\eta - \frac{\sigma\eta - \rho}{\varepsilon})^\nu S_0^\nu = \frac{\sigma\eta}{\alpha},$$

or simply

$$(B_0/K_0)^{1-\alpha} S_0^\nu = cste.$$

One degree of freedom is required to equate the LHS of the above equation with the RHS which is given by the known parameters.

Aghion and Howitt(1998, p.164), by taking  $K_0$  as a function of  $B_0$  and  $S_0$ , abstract from the analysis of transitional dynamics in their model and conclude that the optimal balanced growth path can be reached instantaneously from the initial position. We consider instead the initial value of physical capital, along with the intellectual and natural capital, as given to the central planner. According to this argument in general being on the steady state is unlikely and the economy described by this model lies almost always on its off-balanced path.

**Proposition 22** *Given the initial level of physical, intellectual and natural capital, i.e.  $K_0$ ,  $B_0$  and  $S_0$ , in the economy described by Eqs. (4.1) - (4.6), jumping instantaneously from the initial position to the steady state determined by Eqs. (4.34 - 4.37) is not possible and passing through a transition path is unavoidable.*

### 4.6.3 Dynamics of consumption and physical capital

Consider the following subspace

$$\left\{ \left( \frac{Y}{K}, \frac{C}{K}, n, \frac{R}{S} \right) : \frac{Y}{K}, \frac{C}{K}, \frac{R}{S} \geq 0 \text{ and } 0 \leq n \leq 1 \right\}.$$

To analyze the transitional dynamics of the model in the phase space, one should derive the locus  $\dot{x}_i = 0$  of the components of the vector  $x = (\frac{Y}{K}, \frac{C}{K}, n, \frac{R}{S})^T$ .

From (4.40) the dynamical system describing the behaviour of  $x$  is as follows

$$\begin{aligned} \frac{d}{dt} \left( \frac{Y}{K} \right) &= (\alpha - 1) \left( \frac{Y}{K} \right) \left[ \left( \frac{Y}{K} \right) - \frac{\sigma\eta}{\alpha} \right], \\ \frac{d}{dt} \left( \frac{C}{K} \right) &= \left( \frac{C}{K} \right) \left[ \frac{\alpha - \epsilon}{\epsilon} \left( \frac{Y}{K} \right) + \left( \frac{C}{K} \right) - \frac{\rho}{\epsilon} \right], \\ \frac{dn}{dt} &= n \left[ \frac{C}{K} + \frac{(1 - \alpha)\sigma\eta}{\beta} \left( n + \frac{\nu - 1}{\alpha} \right) \right], \\ \frac{d}{dt} \left( \frac{R}{S} \right) &= \frac{R}{S} \left[ \frac{R}{S} - \frac{C}{K} - \frac{(\alpha - 1)\sigma\eta}{\alpha} \right]. \end{aligned} \quad (4.44)$$

The transition path of the output-capital ratio is depicted in the upper panel of figure 4.2. Besides the origin which is a trivial steady state,  $(Y/K)^* = \sigma\eta/\alpha$  is the stable equilibrium of the first differential equation described in Eq. (4.44).

One sees from (4.21) that  $g_C = (\alpha Y/K - \rho)/\epsilon$  is a linear transformation of  $Y/K$ . The transition path of  $g_C$  is therefore the same as  $Y/K$  except that the former converges to  $(\sigma\eta - \rho)/\epsilon$ . If condition 6 holds, in the long run output, consumption and physical capital grow at a positive (and constant) rate. Sustainability, interpreted as non-negative rate of growth of consumption, requires that  $\alpha Y/K \geq \rho$ . Now if  $(Y/K)_0 < \rho/\alpha$ , then for a finite time  $g_C$  would be negative<sup>11</sup> (see the middle panel of figure 4.2).

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<sup>11</sup>If physical capital depreciates at rate  $\delta > 0$ , then the threshold for output-capital ratio would be  $(\rho + \delta)/\alpha$  instead. Hence the higher the rate of capital depreciation or discount factor, the more likely it is that consumption falls on the transition.

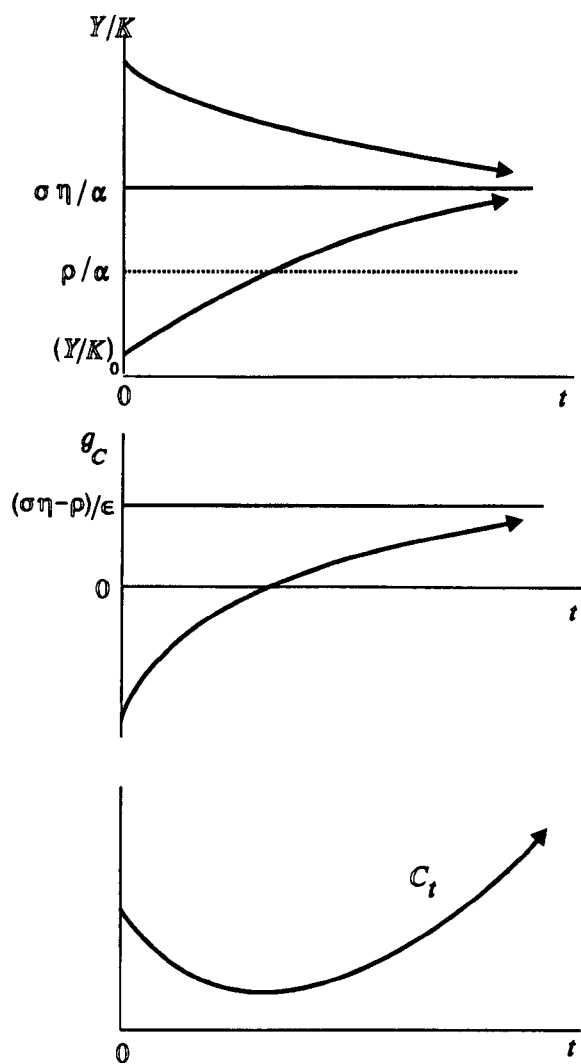


Figure 4.2: Transition path of output-capital ratio (upper panel), consumption growth (middle panel) and time profile of consumption (lower panel)

This implies a U-shaped path for consumption along its time profile; figure 4.2 (lower panel). It falls so long as  $(Y/K)_t < \rho/\alpha$ . Consumption reaches its minimum at  $\arg\{(Y/K)_t = \rho/\alpha\}$ . It then rises when  $Y/K$  passes its threshold toward its steady state. Since the steady state is globally saddle stable, the U-shaped path of consumption is a definite outcome of the model when the initial stock of physical capital is so large that  $(Y/K)_0$  falls below  $\rho/\alpha$ .

The phase diagram of  $(C/K, Y/K)$ , based on the first and second Eqs. of (4.44) is depicted in the upper panel of figure 4.3. As we see  $((Y/K)^*, (C/K)^*)$  is saddle stable. Besides this steady state, and the origin which is the trivial equilibrium, there are two attractors:  $(\sigma\eta/\alpha, 0)$  and  $(\sigma\eta/\alpha, +\infty)$ . The former is intertemporally inefficient and the latter is not feasible. The slope of the saddle path is positive (negative) if  $\epsilon$  is greater (less) than  $\alpha$ .

The stable arm for  $Y/K < \sigma\eta/\alpha$  lies above the  $d(C/K)/dt = 0$  locus. It crosses the 45 degree line from above at  $\widehat{Y/K} = \arg\{Y = C\}$ . Since the saddle path is monotonous, this gives

$$C/K \gtrless Y/K \quad \text{if and only if} \quad Y/K \gtrless \widehat{Y/K}.$$

This, according to (4.22), results in

$$g_K(Y/K) \gtrless 0 \quad \text{if and only if} \quad Y/K \gtrless \widehat{Y/K}.$$

The transitional dynamics of rate of growth of consumption and physical capital, based on these findings, are illustrated in the middle panel of figure 4.3 where we have

1.  $g_K < g_C < 0$  for  $0 < Y/K < \rho/\alpha$ ;
2.  $g_K < 0 < g_C < g^*$  for  $\rho/\alpha < Y/K < \widehat{Y/K}$ ;

3.  $0 < g_K < g_C < g^*$  for  $\widehat{Y/K} < Y/K < \sigma\eta/\alpha$ ;
4.  $0 < g^* < g_C < g_K$  for  $\sigma\eta/\alpha < Y/K$ .

This helps us to draw the U-shaped transitional profile of  $C$  and  $K$  in the lower panel of figure 4.3. Since  $Y/K$  is growing when it is short of its steady state, this graph can be transformed to obtain the time profile of  $C$  and  $K$  too.

The common feature of what we found in figures 4.2 and 4.3, is that *for sufficiently low values of the output-capital ratio, it is optimal for physical capital and consumption to fall for a finite period*. Although the sustainability condition on the steady state guarantees that they all eventually grow at a positive and common rate. The argument suggests that as long as transitional dynamics matter, *one should distinguish between sustainability at the steady state and on the transition path*. The former is necessary but not sufficient for the latter.

#### 4.6.4 Dynamics of resource depletion

From (4.25) we have

$$g_R = -\lambda - C/K,$$

which shows that along the transition path, the rate of decline of resource depletion mirrors the consumption-capital ratio. This gives simply the optimal rate of natural resource depletion along the transition path.

**Proposition 23** *The optimal rate of depletion of natural resource along the transition path is determined by*

$$R_t = R_0 \exp [-\lambda - (C/K)] t.$$

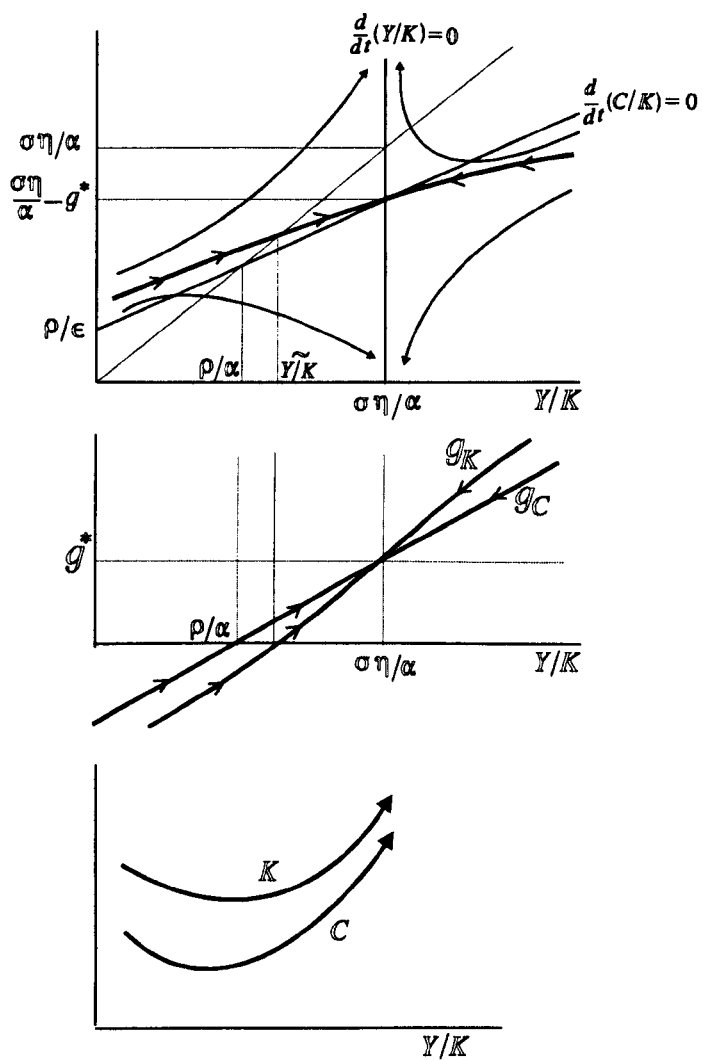


Figure 4.3: Phase diagram of  $Y/K$  and  $C/K$  (upper panel), and the dynamics of consumption and physical capital (middle and lower panels)

From the phase diagram in figure 4.3 we learn that the consumption-capital ratio, along the saddle path, is bounded between the locus  $d(C/K)/dt = 0$  and  $C/K = (C/K)^*$ . This introduces a lower and upper bound on the rate of growth of resource depletion. The result is expressed below.

**Conclusion 10** *When the output-capital ratio is short of its steady state, i.e.  $Y/K < (Y/K)^*$ , the rate of growth of resource depletion is bounded as*

$$\frac{\sigma\eta - \rho}{\epsilon} - \sigma\eta < g_R(t) < -\lambda - \rho/\epsilon.$$

By aid of analytical methods, we cannot explore more details about the dynamics of the rate of resource depletion.

## 4.7 Conclusion

This study takes the Shumpeterian growth model limited by finite supply of exhaustible resources, initiated by Aghion and Howitt(1998, ch.5), as a benchmark. We extend their model to allow for the coexistence of the steady state and transition path, though they only focus on the former. Concerning the steady state, we derive the closed form of the fundamentals of the model and show that constant returns to scale is a sufficient condition for the existence and uniqueness of the steady state. Moreover, we show that the steady state is globally saddle stable where the technology parameters solely determine the rate of convergence along the saddle stable arm toward it.

Incorporation of natural resources, in a first best economy and when the technology of production is Cobb-Douglass, does not affect the long-run rate of growth. This is achieved, however, at the expense of higher employment in the R&D sector, resulting in faster accumulation of knowledge. The higher the rate

of resource utilization or the more important the contribution of natural resource in the aggregate production function, the larger is the gap between the rate of innovation and economic growth. This is obtained because, by assumption, an exhaustible resource does not impose a limitation on the innovation process and the accumulation of knowledge. As a result, the supply of individual brands falls with the rate of utilization of resource, while the total supply of intermediates grows at the balanced rate of growth.

The main contribution of this study lies in the transitional dynamics of the model economy. Our analysis shows that situations exist where it is optimal for both consumption and physical capital to fall for a finite period along the transition path. This happens where the endogenous growth is sustained at the steady state and the sustainability condition, suggested by Aghion and Howitt, is met. Our numerical exercise suggests that the threshold for the occurrence of this situation is when the average productivity of capital is less than a quarter of its steady state. A similar exercise based on other sets of parameters shows that the corresponding threshold might be even higher, suggesting that falling of well-being and the violation of sustainability is likely even when the economy is not far from its steady state.

We believe that transitional dynamics has been overshadowed by the steady state analysis in the literature. Solow(1970)'s words are still alarming where he says "the steady state is not a bad place for the theory of growth to start, but may be a dangerous place for it to end." (p.7)



## Chapter 5

# Concluding Remarks

This study has considered the relation between growth and development on the one side, and exhaustible natural resources on the other. The findings are briefly summarized as follows.

### 5.1 The voracity effect

A country that receives windfalls in the form of a natural resource boom, may react in a cautious or voracious way. In the oil economies, the government is the main recipient body and the fiscal policy is the key channel of the windfall spending. In an economy whose inhabitants are voracious, the size of redistribution caused by a windfall rises more proportionally than the windfall itself.

Taking the voracity effect as granted, we do not explain why the effect takes place, but describe how it proceeds. We attempt to accommodate the issue in two different frameworks. One is a static version that addresses the effect of natural resource abundance on the allocation of human resources and the size of GDP. The other is an established two-sector endogenous growth model in the language

of Lucas. Surprisingly the results are very close, though their structures are far apart. Our findings along this line are summarized in the following:

1. Marginal productivity of lobbying measures the extent of harmful effects of a windfall when the voracity effect is present.
2. A resource boom, in the presence of the voracity effect, changes the reward structure in favour of rent-seeking. By reshaping the incentive structure it diverts labour from productive activities and motivates them toward rent-seeking. The higher the size of boom, or the more severe the propensity to suffer from voracity, the greater is the extent of diversion.
3. An increase in government revenues, financed by resource rents, lowers the provision of public productive services if the voracity effect is present.
4. If lobbying is the only motive behind fiscal transfers, the voracity effect does not appear when rent-seeking intensity is low. In particular, if marginal productivity of lobbying changes in the same direction as rent-seeking intensity, the voracity effect takes place only for high levels of rent-seeking. The latter mechanism is more likely to operate if resource rent and rent-seeking intensity have separate effects on the level of fiscal transfers.
5. By distinguishing between the income effect of boom and its effect on the choice of occupation, we specify to what extent the *horror* aspects of booms can be represented by the voracity effect. This can be described as follows:
  - (a) We derive a sufficient condition for the case where a resource boom, in contrast to its positive direct effect, is counterproductive. This happens when the voracity effect operates or when the extent of diversion

of labour induced by a boom is large enough to compensate for its positive income effect.

(b) Three factors account for the negative level effect of a boom: the extent of voracity; the share of transfers in fiscal expenditures and the level of resource intensity. Whereas the first and the second elements magnify the adverse welfare effect of boom, the third factor mitigates it.

6. A representative-agent framework can capture the main aspects of the voracity effect. To generate the voracity effect, we do not necessarily need to address redistribution among the agents and go beyond the representative-agent framework. This corresponds to the occurrence of the situation where the economy reacts to a relative abundance of physical to human capital by more consumption. A resource boom in the form of a sudden increase of physical capital therefore leads to a lower steady state.
7. We provide a necessary and sufficient condition for the appearance of the voracity effect. This is done by introducing voracity as an extreme case of being impatient, which clarifies the link between the voracity and the consumption-smoothing effect.
8. We show that the occurrence of voracity is a sufficient condition for development failure of booming economies. The adverse effects of the resource boom on the allocation of talent, the size of GDP and rate of growth of the economy may emerge even when voracity is not operative.

## 5.2 Development failure of oil economies

Although the voracity effect is a well designed concept which addresses the harmful aspects of natural resource booms, it does not fully capture all of the boom effects. We classify the impact of booms on the allocation of labour between productive and unproductive activities, level and composition of GDP, based solely on the characteristics of the influence function. This includes situations where the size of fiscal claim does not exceed the windfall, but the harmful impacts of booms are still present.

Regardless of the volatility of the natural resource rent, the mere endowment of natural resources has some important impacts on the general performance of the economy. The findings can be summarized along this line as follows:

1. The mere access to the natural resource rent can justify the occurrence of the no-activity equilibrium where the labour force are rent-seekers and the extraction of natural resource is the only productive activity.
2. If fiscal transfers rise proportionally with oil rent, the size of the latter and hence the degree of resource intensity has no effect on the occupational choice. In other words, the nonlinear effect of resource rent on fiscal transfers is the channel of transmission of the extent of natural resource abundance on rent-seeking intensity. This of course implies that if fiscal policy in a booming economy is such that fiscal transfers grow proportionally with resource rents, the boom does not affect the allocation of human resources.
3. There is an association between rent-seeking activities and oil abundance. Windfall spending through fiscal channels, political distribution of rents,

and the extent of influence of lobbying on windfall spending strengthen the link. The voracity of effective windfall claimants is a sufficient but not a necessary condition for a resource boom to induce diversion of human resources.

4. We derive a necessary and sufficient condition for a resource boom to lower the level of GDP. We also find that an oil boom is welfare worsening if the voracity is in effect and the propensity to suffer from the voracity exceeds the size of subsidy to producers in units of the final good.
5. The endowment of plant and equipment in the oil economies is highly unbalanced with respect to their stock of human resources. This suggests that oil windfalls are invested mainly in physical capital rather than education.
6. When the consumption-smoothing effect is not strong enough, there is a monotonic association between the extent of imbalance between physical and human capital and the rate of output growth in the vicinity of the steady state. The lower the rate of economic growth, the more scarce is the relative human capital. This, in conjunction with our previous findings, suggests an explanation for the local adverse growth effect of oil booms. This also is proved to be useful to partially explain the poor growth performance of oil abundant economies.
7. Our empirical findings on the extent of imbalance between physical and human capital in oil economies, bridges our results on U-shaped path of consumption and physical capital to the macroeconomic performance of oil economies. This provides an approximation for the high but declining trend of consumption and physical capital in the underlying economies.

The more biased the spending of oil rents toward the physical capital accumulation away from education enhancing, the further the boom displaces the economy from the steady state and the more likely the booming economy is to live beyond its means. This is attributable to a high level of consumption which is not sustainable, and a high level of physical capital which is not consistent with the current level of skills and knowledge.

### **5.3 Transition dynamics of consumption and physical capital**

Lucas' key assumption that education depends only on the pre-accumulated stock of human capital and the amount of time devoted to learning, identifies a cluster of multi-sector endogenous growth models where only one type of capital can be used across other sector(s). Within this class we explore the transitional dynamics of consumption and physical capital. This leads to the following results:

1. In the two-sector Lucas growth model,
  - (a) when human capital is relatively scarce, consumption and physical capital may exhibit U-shaped trends during their convergence path toward their steady state. They firstly fall for a finite period and then rise toward their balanced growth path.
  - (b) the sequence of falling and rising of consumption and physical capital do not match. There exists a situation where the rising of consumption coincides with the falling of capital. In contrast with the findings of Caballé and Santos(1993), this distinguishes the dynamics of the Lucas-type growth models from overaccumulation of capital in the

Ramsey economy. Moreover the sequence of stages differs substantially from those of Ramsey.

(c) rates of growth of consumption and physical capital, like the Ramsey model, change in the same direction during the transition. The difference is that their signs do not change at the same time.

(d) imposing the baseline parameters in the literature, one can observe that the falling period happens over a reasonably long period of time.

2. Our findings about the dynamics of consumption and physical capital and the local dynamics of output growth is applicable to the whole class of the multi-sector endogenous growth models that share the Lucas assumption. This in particular includes the established R&D-based growth models of Romer and Aghion-Howitt.

3. Growth failure of oil economies and their after-boom effects, can be addressed in a more explanatory way by a two-sector Lucas-type rather than by a Ramsey model. In particular, some regularities of oil economies are in favour of the former model which is able to address the imbalance effect.

## **5.4 Sustainable development versus endogenous growth**

Our findings on the transitional dynamics of consumption and physical capital in the Lucas-type multi-sector growth models show that the sustainability condition proposed by Aghion and Howitt(1998, ch.5) is a necessary but not sufficient condition for the non-declining path of consumption and physical capital along their transition path.

In an economy that meets the sustainability condition, in the long run consumption, capital and output all grow at a positive and common rate. In the short and medium term however, the U-shaped path of consumption and physical capital indicates that sustainability is violated along the transition path.

## **5.5 Growth limited by exhaustible resources**

We consider an innovation-based three-sector endogenous growth model limited by finite supply of nonrenewable natural resources. The model is taken from Aghion and Howitt(1998, ch.5) where the authors address the advantages of the creative destruction approach to deal with sustainability issues in comparison with other growth models.

Our study offers the following contributions to the Aghion-Howitt model.

1. Given the initial portfolio of the wealth of a country, including the stock of physical and natural capital and the level of knowledge, we show that the transitional dynamics exists. This means that the steady state analysis only identifies the behaviour of the economy in the long run, whereas the economy spends most of its time along the transition path.
2. The Aghion-Howitt model shares the similar reduced form with the Lucas growth model and thus our findings about the dynamics of consumption and physical capital and local dynamics of output growth in the latter model are applicable to the former model too.
3. The dynamics of rate of resource depletion is further explored. It mirrors consumption-capital ratio along the stable saddle path.



4. Having examined the validity of our findings about the U-shaped path of consumption and physical capital in the Aghion-Howitt model, we show that the debate about the linkage between sustainable development and endogenous growth is applicable and is even clearer if one takes the limitations imposed by exhaustible natural resources into account.

We also examine the welfare cost of running out of exhaustible resources that are essential inputs of aggregate production. We take Weitzman(1999)'s model as a benchmark where in a general setting, he derives the forgone consumption resulting from exhaustion of minerals on the basis of time trend of mineral rents. We apply his logic to a CES framework and find that the elasticity of production with respect to the minerals measures this welfare indicator properly..

## Appendix A

# Appendix to Chapter 2

### A.1 The expected prize in a multi-grant contest

Plugging from definition of  $P_i(j)$  into (2.5) gives

$$\tilde{V}_i = R \sum_{j=0}^k \frac{j}{k} \binom{k}{j} \pi_i^j (1 - \pi_i)^{k-j},$$

where  $\pi_i = x_i/X$ . From basic combinatorics we know

$$\frac{j}{k} \binom{k}{j} = \binom{k-1}{j-1}.$$

Putting this into the above expression leads to

$$\begin{aligned} \tilde{V}_i &= R \sum_{J=-1}^K \binom{K}{J} \pi_i^{J+1} (1 - \pi_i)^{K-J} \\ &= R \pi_i \sum_{J=0}^K \binom{K}{J} \pi_i^J (1 - \pi_i)^{K-J}. \end{aligned}$$

where by binomial expansion, the summation equals one.

## A.2 Proofs

This section provides proof of the theorem-type statements in Chapter 2 whose proofs are not appeared in the text itself.

**Lemma 1** From (2.2),  $R < Q$  leads  $T < G$  which applying (2.3) and (2.1) gives the result.

**Lemma 3** Assumptions imposed on  $R$  are sufficient for  $\phi > 0$  when  $n \in (0, 1]$ . Moreover we have

$$\phi'(n) = \phi(n)^2 \left[ \frac{2(1 - \tau AL) + \tau ALn}{(1 - \tau)AL^2n^3} \right],$$

which is positive. By plugging from  $\phi$ , the RHS can be rewritten as

$$n [(1 - \tau)AL]^2 \{n^2(1 - \tau)AL + [1 - \tau AL(1 - n)] / L\}^{-2},$$

which tends to zero when  $n \rightarrow 0$ . The last part is trivial.

**Lemma 5** Differentiating from  $V$  and  $W$  in Eqs. (2.13) and (2.14) with respect to  $Q$ , gives the result.

**Lemma 6** By differentiating from (2.14) and (2.13) with respect to  $Q$ ,  $W_Q \leq V_Q$  is equivalent with  $(1 - \tau)A(1 - R_Q) / [1 - \tau AL(1 - n)] \leq R_Q / L^2 n^2$  where the latter inequality can be rewritten as

$$\frac{(1 - \tau)A}{1 - \tau AL(1 - n)} \leq \left[ \frac{(1 - \tau)A}{1 - \tau AL(1 - n)} + \frac{1}{L^2 n^2} \right] R_Q.$$

This, by lemma 1, gives  $\{1 + [1 - \tau AL(1 - n)] / (1 - \tau)AL^2n^2\} R_Q \geq 1$  where lemma 1 once again ensures that the expression within the bracket is positive.

# Appendix B

## Appendix to Chapter 3

### B.1 The algorithm for numerical simulation

This part explains an algorithm, based on the time elimination method, which is used in sections 3.4 and 3.5 for simulating the off-balanced path of the Lucas model described in section 3.3. The method was first introduced by Mulligan and Sala-i-Martin(1993) into the field.

Consider a Lucas economy defined by Eqs. (3.1) - (3.4) with known parameters  $A, B, \alpha, \rho, \delta, \sigma$ . The off-balanced path of this economy is a sequence of points  $(z_i, \chi_i, u_i)$  that are generated through the following algorithm.

1. Initialization:

(a) Let  $\epsilon > 0$  be a small deviation from the steady state<sup>1</sup> defined by (3.7),

$\Delta t$  the step size and  $T \leq -2 \ln(\tilde{z}/\epsilon - 1)/\lambda$  the length of simulation.

(b) Let  $z_0 = \tilde{z} - \epsilon$ ,  $\chi_0 = \tilde{\chi} - \epsilon v_0$  and  $u_0 = \tilde{u} - \epsilon w_0$  define the initial

---

<sup>1</sup>Since  $z$  approaches asymptotically to  $\tilde{z}$ , it never reaches it in practice. We thus displace  $\tilde{z}$  by a small disturbance  $\epsilon$ . For  $\epsilon < 0$ , the algorithm generates the right saddle path when  $z > \tilde{z}$ .

condition in the vicinity of the steady state where  $v_0$  and  $w_0$  are the second and third components of the stable eigenvector,  $\lambda$  defined in Eq. (3.12).

(c) Set  $i = 0$ .

2. Going backward from the steady state, generate the next point on the saddle path as follow:

$$(a) \Delta z_i = (1 - \alpha)z_i(\tilde{z} - z_i)\Delta t;$$

$$(b) z_{i+1} = z_i - \Delta z_i;$$

$$(c) \chi_{i+1} = \chi_i - v_i \Delta z_i;$$

$$(d) u_{i+1} = u_i - w_i \Delta z_i,$$

where

$$v_i = \frac{(\alpha/\sigma - 1)z_i + \chi_i - \rho/\sigma + (1 - 1/\sigma)\delta}{(\alpha - 1)z_i - \lambda} \cdot \frac{\chi_i}{z_i},$$

$$w_i = \frac{-\lambda + Bu_i - \chi_i}{(\alpha - 1)z_i - \lambda} \cdot \frac{u_i}{z_i}.$$

3. If  $i \geq T$  algorithm ends. Otherwise set  $i = i + 1$ , Go to step 2.

The sequence  $\{z_i, \chi_i, u_i\}_{i=1}^T$  presents the off balanced path of the economy for  $z < \tilde{z}$ .

## B.2 Solution of the model with externality

This appendix gives a solution of the Lucas growth model with externality presented in section 3.6. This, also in the absence of externality, i.e. where  $\gamma = 0$ , produces the solution of the model in section 3.3. Moreover with  $A = 1$ ,  $H = B$ , and  $\gamma = 1 - \alpha - \beta$ , the results are also applicable to sections 4.4 and 4.5.

The current value Hamiltonian of the model adopted by the central planner is given by

$$J = \frac{C^{1-\sigma} - 1}{1-\sigma} + \nu [AK^\alpha(uH)^{1-\alpha}\overline{H}^\gamma - C - \delta K] + \mu[B(1-u)H - \delta H],$$

where  $\nu$  and  $\mu$  are the costate variables. The term in the first set of brackets equals  $dK/dt$ , and the term in the second set of brackets equals  $dH/dt$ . The former in the competitive economy is replaced by  $(r - \delta)K + wHu - C$  where  $r$  and  $w$  are defined in (3.17).

The first-order conditions,  $\partial J/\partial C = 0$  and  $\partial J/\partial u = 0$ , lead respectively to

$$C^{-\sigma} = \nu, \tag{B.1}$$

$$\nu(1 - \alpha)Y/u = \mu BH, \tag{B.2}$$

where in (B.1) the isoelastic form of the instant utility from (3.1) is taken into account. Condition (B.2) in the competitive economy is simply  $\nu w = \mu B$ .

Condition  $d\nu/dt = \rho\nu - \partial J/\partial K$  implies

$$\dot{\nu}/\nu = \rho - \alpha z + \delta, \tag{B.3}$$

where the right hand side in the decentralized version is  $-(r - \rho - \delta)$ .

Condition  $d\mu/dt = \rho\mu - \partial J/\partial H$  implies

$$\dot{\mu}/\mu = \rho - (1 - \alpha + \gamma)Y/H - B(1 - u) + \delta, \tag{B.4}$$

in which the consistency condition  $H = \overline{H}$  has been imposed. In the competitive economy however, owing to lack of consideration of the human capital externality, the second term of the right hand side is  $wu$ .

The transversality conditions are

$$\lim_{t \rightarrow \infty} e^{-\rho t} \nu_t K_t = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu_t H_t = 0.$$

Differentiating Eq. (B.1) with respect to time, in conjunction with (B.3), gives the Euler equation for consumption growth

$$g_C = (\alpha z - \rho - \delta) / \sigma, \quad (B.5)$$

Moreover (3.3) gives the rate of growth of physical capital as,

$$g_K = z - \chi - \delta, \quad (B.6)$$

The growth rate of  $\chi$  can then be determined from Eqs. (B.5) and (B.6) which is valid for both decentralized and centrally planned economies:

$$g_\chi = g_C - g_K = (\alpha / \sigma - 1) z + \chi - (\rho + \delta) / \sigma - \delta. \quad (B.7)$$

If one now substitutes for  $\nu / \mu$  from (B.2) into (B.3), the result is

$$\begin{aligned} \dot{\nu}^o / \nu &= -(B - \rho - \delta) - \gamma B u / (1 - \alpha), \\ \dot{\nu}^e / \nu &= -(B - \rho - \delta), \end{aligned}$$

where the superscripts  $o$  and  $e$  refer to the optimal path and competitive equilibrium respectively. On the other hand, differentiation with respect to time from (B.2) gives another expression for the rate of growth of the shadow price of human capital as:

$$\dot{\nu} / \nu = \rho + \delta(1 - \alpha) - \alpha \chi - \alpha g_u + (\gamma - \alpha)[B(1 - u) - \delta].$$

By setting equal the right hand side of two expressions obtained for  $\dot{\nu}/\nu$ , one gets the rate of change of work effort along the optimal path and in the competitive equilibrium respectively as

$$g_u^o = -\lambda - \chi + Bu(1 - \alpha + \gamma)/(1 - \alpha), \quad (\text{B.8})$$

$$g_u^e = -\lambda - \chi + Bu(\alpha - \gamma)/\alpha, \quad (\text{B.9})$$

where  $\lambda = -[B(1 - \alpha + \gamma) - \delta\gamma]/\alpha$ .

If we differentiate Eq. (3.15) with respect to time, in conjunction with the above results, we get, after simplifying,

$$g_z^o = g_Y^o - g_K = -\lambda + (\alpha - 1)z, \quad (\text{B.10})$$

$$g_z^e = g_Y^e - g_K = -\lambda + (\alpha - 1)z - Bu\gamma/\alpha. \quad (\text{B.11})$$

Equations (B.10), (B.7) and (B.8) form a system of three growth differential equations in the variables  $z$ ,  $\chi$  and  $u$ , where the state variable  $z$  begins at some value  $z(0)$ . In the decentralized economy the first and third equations are replaced with (B.11) and (B.9).

Along a balanced growth path, by definition, rates of growth of  $C$ ,  $K$  and  $H$  are constant and  $u$  does not change. From (B.5), we conclude that  $z$  at the steady state is constant meaning that in the long run  $Y$  and  $K$  are proportional. This, in conjunction with (B.6), implies that at the steady state  $\chi$  is also constant meaning that  $C$  and  $K$  grow in the long run at a common rate too. We call this the balanced rate of growth  $\tilde{g} \equiv \tilde{g}_C = \tilde{g}_K = \tilde{g}_Y$ , where a tilde refers to the steady state. If we differentiate Eq. (3.15) with respect to time and substitute from above, it results in:

$$\tilde{g} = (1 + \gamma/(1 - \alpha)) \tilde{g}_H. \quad (\text{B.12})$$



The steady state of the centrally planned economy can be found readily by setting the three growth Eqs. (B.10), (B.7) and (B.8) to zero. This results in

$$\begin{aligned}\tilde{z} &= [B(1 - \alpha + \gamma) - \delta\gamma]/[\alpha(1 - \alpha)], \\ \tilde{\chi} &= B/\alpha - (B - \rho - \delta)/\sigma - \delta - (B - \delta)(1/\sigma - 1/\alpha)/(1 - \alpha), \\ \tilde{u} &= 1 - \{\rho - [\lambda - (1 - \alpha)(\rho - \delta)]/[\sigma(1 - \alpha + \gamma)]\}/B.\end{aligned}$$

For the decentralized economy one should set the Eqs. (B.11), (B.7) and (B.9) to zero to obtain

$$\begin{aligned}\tilde{z} &= B/\alpha + [(B - \rho - \delta)\gamma/\alpha]/[\sigma(1 - \alpha + \gamma) - \gamma], \\ \tilde{\chi} &= \tilde{z} - \tilde{g} - \delta, \\ \tilde{u} &= 1 - [(1 - \alpha)(B - \rho - \delta)/B]/[\sigma(1 - \alpha + \gamma) - \gamma] - \delta/B.\end{aligned}$$

The corresponding steady state growth rate of  $Y$ ,  $C$  and  $K$  in the centrally planned and decentralized economy are respectively:

$$\begin{aligned}\tilde{g}^o &= \frac{(B - \rho)(1 - \alpha + \gamma)}{\sigma(1 - \alpha)} - \frac{\delta}{\sigma}, \\ \tilde{g}^e &= \frac{B - \rho - \delta}{\sigma - \gamma/(1 - \alpha + \gamma)}.\end{aligned}$$

These two expressions, taking into account (B.12), give the long run rate of human capital accumulation at a lower rate equal to

$$\begin{aligned}\tilde{g}_H^o &= \frac{(B - \rho - \delta)(1 - \alpha) + \gamma(B - \rho)}{\sigma(1 - \alpha + \gamma)}, \\ \tilde{g}_H^e &= \frac{(1 - \alpha)(B - \rho - \delta)}{\sigma(1 - \alpha + \gamma) - \gamma}.\end{aligned}$$

The external effect, therefore, induces more rapid physical than human capital accumulation in such a way that, overall the economy grows faster.

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